Mathematical Sciences Research Institute
Berkeley, CA 94720

May 20, 1994

Dear Dennis:

I know I am bombarding you with letters. But I am so pleased at what happened this morning that -- here goes.

The forms $f = x^2 + 3y^2 + 5z^2$ and $g = x^2 + 2y^2 + 2yz + 8z^2$ form a genus. I am far from knowing what integers they represent (though I have some computational data). But elsewhere 4n's and (4n + 1)'s have behaved better so I tried. And here's the point: I successfully imitated you.

Theorem: f represents all eligible 4n's and (4n + 1)'s.

Proof. $f(x, w + 2z, w) = g(x, 2w + z, z)$. Suppose $g(u, v, w) = B$. I repeat verbatim the proof that f represents B.

Now let's go the other way. We hit a snag: f lacks the symmetry of $x^2 + y^2 + 7z^2$. Lo and behold, the same thing happens as for discriminant 11: 4n works and 4n + 1 doesn't. As for the failure, g doesn't represent 5. But that's unconvincing: 5 divides the discriminant. (I recall that it looks possible that $x^2 + y^2 + 11z^2$ represents all ~~numbers~~ (4n+1)'s prime to 11.) But g also fails to represent 53; that's convincing.

Theorem. g represents all eligible 4n's.

Proof. $f(u, v, w) = A$, divisible by 4. If v and w have the same parity, done as before. Now either u, v, w are all even, and that's fine, or just one is even. The even one can't be v (check mod 4). If it's u that's good. Suppose it's w. Then I can change the odd numbers u and v to even ones (this trick occurs in Jones's thesis and may be older -- I used it in my last letter.)

Done.

Regards,
Kop