(4)

Oct. 12/97

<u>A unified discussion of 1, 4, 9n (Jagy) and 1, 4, 36 (J-P).</u>

For 1, 4, 36 the problem is to represent eligible $4n+1$'s and this looks after 1, 4, 9 on $4n+1$ (and, incidentally, also 1, 1, 36 on $4n+1$; these two forms are nonregular, whereas 1, 4, 36 is). Then the problem for 1, 4, 9 is to represent every eligible $4n+2$ except 2.

If the target A is $\equiv 1\ (3)$ the discussion is so easy that I omit it. So assume $A \equiv 2\ (3)$. Now there are two tricks. 1) Write $A = x^2 + 2y^2 + 2z^2$ (instead of $x^2+y^2+z^2$). 2) There are two cases: I x, y div. by 3, z not. II All three prime to 3 the trick is to use the "rediscovered lemma" to switch I to II. There is the except $x = y = 0$. This doesn't occur for 1, 4, 36 since the target is odd; for 1, 4, 9 we get the exception 2.

Then we polish off II. We rewrite A as $x^2 + (y+z)^2 + (y-z)^2$. (Note that one of $y \pm z$ is div by 3). For 1, 4, 36 we have that y and z have the same parity. For 1, 4, 9 we have that x is even. Done