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Brandt, Heinrich; Intrau, Oskar
Tabellen reduzierter positiver ternärer quadratischer Formen. (German)
Abh. Sächs. Akad. Wiss. Math.-Nat. Kl. **45** 1958 *no. 4* 261 *pp.*

The authors use the notation

$$f = a_1{x_1}^2 + a_2{x_2}^2 + a_3{x_3}^2 + a_4x_2x_3 + a_5x_3x_1 + a_6x_1x_2$$

for ternary quadratic forms, and

$$d = a_1{a_4}^2 + a_2{a_5}^2 + a_3{a_6}^2 - a_4a_5a_6 - 4a_1a_2a_3$$

is the formula for the discriminant. A form in which the a's are integers is called "primitive" if 1 is the g.c.d. of the a's. This table lists all reduced primitive positive ternary quadratic forms with integral coefficients with discriminants from -2 to -1000. There are over 36,000 forms listed. [Cf. the shorter tables of the reviewer, Nat. Res. Council Bull. no. 97 (1935)].

Two forms are of the same genus ("verwandt") if one may be taken into the other by a non-singular linear transformation with rational coefficients. The fundamental discriminant ("Stammdiskriminante") of a genus is the least discriminant among the forms of the genus with integral coefficients.

The adjugate form of f has the coefficients

$$\begin{gathered}{a_4}^2 - 4a_2a_3, \quad {a_5}^2 - 4a_3a_1, \quad {a_6}^2 - 4a_1a_2, \\ 4a_1a_4 - 2a_5a_6, \quad 4a_2a_5 - 2a_4a_6, \quad 4a_3a_6 - 2a_4a_5.\end{gathered}$$

The author denotes by I_1 the g.c.d. of these coefficients and defines I_2 by ${I_1}^2I_2 = 16d$. Two forms with the same invariants I_1, I_2 are said to be of the same order and I is defined by $I = I_1I_2/16$.

The basic conditions for a reduced form are

$$0 < a_1 \le a_2 \le a_3, \quad |a_6| \le a_1, \quad |a_5| \le a_1, \quad |a_4| \le a_2,$$

and, in case a_4, a_5, a_6 are all negative,

$$|a_4 + a_5 + a_6| \le a_1 + a_2.$$

These do not define in all cases a unique reduced form and the author merely sketches further considerations leading to unicity. He is not aware of or chooses to disregard the complete conditions obtained laboriously by L. E. Dickson [*Studies in the theory of numbers*, Chicago, 1930, Chap. IV].

In the table, forms for each discriminant are classified according to order and genus and the following invariants given: the number of automorphs, the number of forms in each genus, the prime factors of the discriminant, I_1, I_2, I and the related invariants of Minkowski, the fundamental discriminant and the characters.

This is a monumental piece of work and should be of great service to those working with quadratic forms.

Reviewed by *B. W. Jones*