Strodt's approach to differential equations in the complex domain

In [1] Strodt deals with principal solutions to standard algebraic ODEs in the complex domain, where *principal* means "slowest growing" and *standard* means having coefficients either asymptotic to a logarithmic monomial or trivial (approaching zero faster than any negative power of z).

Strodt introduces an asymptotic notion which is stronger than the usual in the sense that it involves a condition on the derivatives (automatic over the reals as a consequence of l'Hôpital's rule with one function in the quotient being the n-th product of successive iterated logs). He obtains principal solutions using an algorithm based on the Newton-Ritt polygon.

For first order equations he uses one of principal monomials to transform the equation into normal form and with many estimates shows uniform convergence of an iterative scheme in small sectorial regions.

Subsequently the solutions are extended to larger sectors. It turns out that the zeros of a certain indicial functional of the angle split up the domain (the complement of the negative real axis) into sectors, where one gets either a unique or a one parameter family of solutions which tend to 0 smoothly.

In [2] Strodt reworks some of [1] and develops an asymptotic factorization theory to extend results beyond the initial task of finding approximate solutions to n-th order.

The program:

(i) reduce the equation to first order by asymptotic factorization

(ii) find approximate solutions of smallest growth in the form of logarithmic monomials by a generalization of Newton's polygon

(iii) use approximate solutions (M) to reduce the original equation to normal form, i.e. write the exact solution as $W = M(1+\alpha)$, where $\alpha \to 0$ and deal with the equation for α — a first order algebraic ODE solved by iterated variation of parameters

(iv) apply an iteration scheme to obtain actual solutions asymptotically represented by the above monomials in small sectors

(v) continue solutions analytically to natural barriers determined by the zeros of an indicial function for the angle

References:

[1] W. Strodt, *Contributions to the asymptotic theory of ordinary differential equations in the complex domain,* Mem. Amer. Math. Soc. **13** (1954)

[2] W. Strodt, *Principal solutions of ordinary differential equations in the complex domain,* Mem. Amer. Math. Soc. **26** (1957)