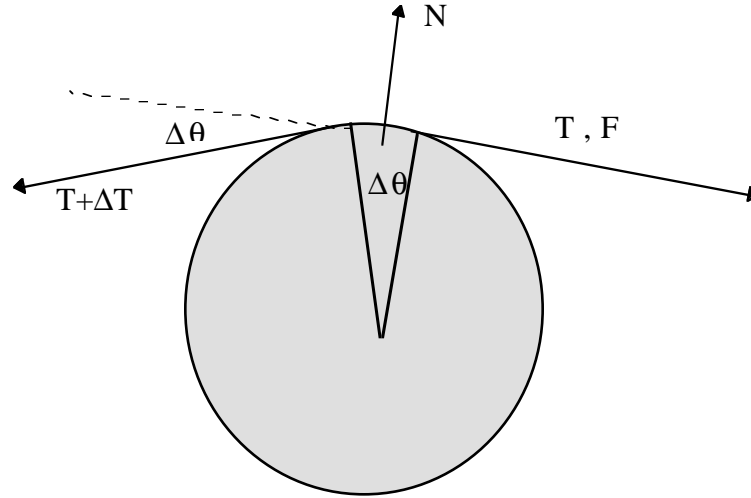


Lab 8.

Cable in Contact with a Rough Surface.

Here we consider the mathematics of a rope or cable passing over a rough surface. You may have already come across the phenomenon of wrapping a rope around a tree or a post, for example, and producing a situation in which huge loads can be held. You may have seen tree cutters use this to lower branches.

Consider the situation as depicted in the sketch.



We will now balance the forces acting on a small piece of the cable Δs .

First we balance the forces in the tangential direction:

$$T + F\Delta s = (T + \Delta T) \cos(\Delta \theta).$$

If we use the tangent line approximation for $\cos(\Delta \theta)$, which is simply $\cos(\Delta \theta) \approx 1$, we have,

$$T + F\Delta s \approx (T + \Delta T).$$

Thus on letting $\Delta s \rightarrow 0$, we derive,

$$(i) \quad \frac{dT}{ds} = F.$$

Secondly we balance the forces in the direction perpendicular to the tangent, that is to say along the normal,

$$N\Delta s = (T + \Delta T) \sin(\Delta \theta).$$

If we use the tangent line approximation for $\sin(\Delta \theta)$, which is $\sin(\Delta \theta) \approx \Delta \theta$, we have,

$$N\Delta s \approx (T + \Delta T)\Delta \theta,$$

and so if we let $\Delta s \rightarrow 0$, we derive,

$$(ii) \quad N \frac{ds}{d\theta} = T.$$

However if the rope is on the point of slipping then

$$(iii) \quad F = \mu N,$$

where the symbol μ (pronounced *mu*) is the coefficient of friction. The larger this coefficient is the rougher the surface.

Now work the following problems.

(a) By eliminating F and N from the equations (i), (ii) and (iii) show that

$$(iv) \quad \frac{dT}{d\theta} = \mu T.$$

This is an old friend, it shows that T increases exponentially with θ .

(b) Now let's work a problem of a rope wrapped around a post.

Determine how much initial tension is needed, that is to say the tension in the rope when it first meets the post, in order for it to support a one ton weight if the rope is wrapped around the post two times. Let the coefficient of friction be 0.5.

(c) Recall that we said that equation (iii) holds only if the rope is on the point of slipping. In general the friction coefficient μ will be a function. In our problem let's suppose that the friction depends on the tension in such a way that

$$\begin{aligned} \mu &= T / 20, \text{ for } 0 \leq T \leq 10, \\ &= 1/2, \text{ for } 10 < T. \end{aligned}$$

Redo problem (b) with this function replacing the constant value we assumed above.