

Lab 9.

Central Orbits.

If we make some simplifying assumptions it can be shown that, relative to the sun, the orbits of the planets and some comets are ellipses. The equations of those ellipses can be written in polar form as,

$$r(\theta) = \frac{a(1 - e^2)}{1 - e \cos(\theta)}.$$

In this formula e is the *eccentricity* and a is the *semimajor axis* of the ellipse. The eccentricity is a number between 0 and 1. If it is 0 then the orbit is a circle. If it is close to 1 the orbit is a long stretched out ellipse. The eccentricity of the earth's orbit around the sun is 0.017, while, for example the eccentricity of Halley's comet is 0.969. Thus the earth's orbit is very nearly circular, while the orbit of Halley's comet is a long skinny ellipse. If we measure distances in astronomical units (one AU is 10^{11} m) and time in years then the period of the orbit T is related to the semimajor axis by the formula,

$$T^2 = a^3.$$

This is Kepler's third law.

The earth's semimajor axis is very nearly 1 and the period of Halley's comet is 76.03 years.

(a) Use this information to simultaneously plot the orbits of the earth and Halley's comet. Make sure you constrain you plot.

(b) Determine how long Halley's comet is within the earth's orbit. To do this first compute the angular momentum h ,

$$h = 2\pi\sqrt{a(1 - e^2)}.$$

Then use Kepler's second law which says that the radius vector sweeps out equal areas in equal times. In other words the time taken to go from one point in the orbit to a second is proportional to the area swept out by the radius vector in moving from the first point to the second, in fact,

$$t = \text{Area} \times 2 / h.$$

Finally the area traced out by a radius vector is $\frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$.

Evaluate this integral by the trapezoid rule with 50 subintervals, that is take the average of the leftsum and the rightsum.