MAT 3623.001 Mid 3 Sp '16

(1) Since f is even, its Fourier Series has no sines.

$a_0 = \text{ave}(f) = 1$ For $n \geq 1$ $a_n = \frac{1}{2}\int_{-2}^{2} f(t)\cos\left(\frac{n\pi t}{2}\right)dx = \int_0^2 (2-t)\cos\left(\frac{n\pi t}{2}\right)dx$

$2-t \ \oplus \ \cos\left(\frac{n\pi t}{2}\right)$

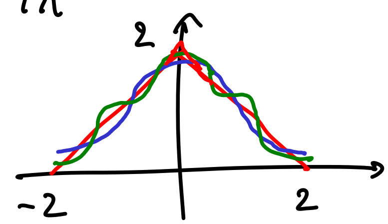
$-1 \ \ominus \ \frac{2}{n\pi}\sin\left(\frac{n\pi t}{2}\right)$

$0 \ \ -\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi t}{2}\right)$

$$= \left[\frac{2(2-t)}{n\pi}\sin\left(\frac{n\pi t}{2}\right) - \frac{4}{n^2\pi^2}\cos\left(\frac{n\pi t}{2}\right)\right]_0^2 = \frac{4}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$a_1 = \frac{8}{\pi^2}, \quad a_2 = 0, \quad a_3 = \frac{8}{9\pi^2}$$

$$\boxed{f = 1 + \frac{8}{\pi^2}\cos\left(\frac{\pi t}{2}\right) + \frac{8}{9\pi^2}\cos\left(\frac{3\pi t}{2}\right) + \ldots}$$

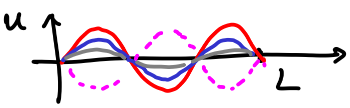


(2) $u_{tt} = c^2 u_{xx}$. Let $u = F(x)G(t)$. $FG'' = c^2F''G$

$\frac{F''}{F} = \frac{G''}{c^2G} = -k^2$, $F'' + k^2F = 0$, $G'' + c^2k^2G = 0$

If $F = \sin\left(\frac{3\pi x}{L}\right)$, $k = \frac{3\pi}{L}$, so $G'' + \left(c\frac{3\pi}{L}\right)^2 G = 0$

So let $\boxed{u = \sin\left(\frac{3\pi x}{L}\right)\cos\left(c\frac{3\pi}{L}t\right)}$



(3) $\nabla^2 u = 0 \Rightarrow u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$

Let $u = F(r)G(\theta)$. Then $F''G + \frac{1}{r}F'G = -\frac{1}{r^2}FG''$

$\frac{r^2F''}{F} + \frac{rF'}{F} = -\frac{G''}{G} = n^2$, $r^2F'' + rF' - n^2F = 0$, $G'' + n^2G = 0$

If $G = \sin(3\theta)$, $n = 3$, so $r^2F'' + rF' - 9F = 0$

Try $F = r^\alpha$: $r^2\alpha(\alpha-1)r^{\alpha-2} + r\alpha r^{\alpha-1} - 9r^\alpha = 0$

$\alpha^2 - \alpha + \alpha - 9 = 0$, $\alpha = \pm 3$, since r^{-3} blows up at $r=0$, $\alpha = 3$

Since constants satisfy the eq., by linearity

$\boxed{u = 23 - \frac{2}{5^3}\sin(3\theta)r^3}$ is the desired solution.

1

(%i1)
```
f:2-abs(x);
plot2d(f,[x,-2,2])$
```

(%o1) $2-|x|$

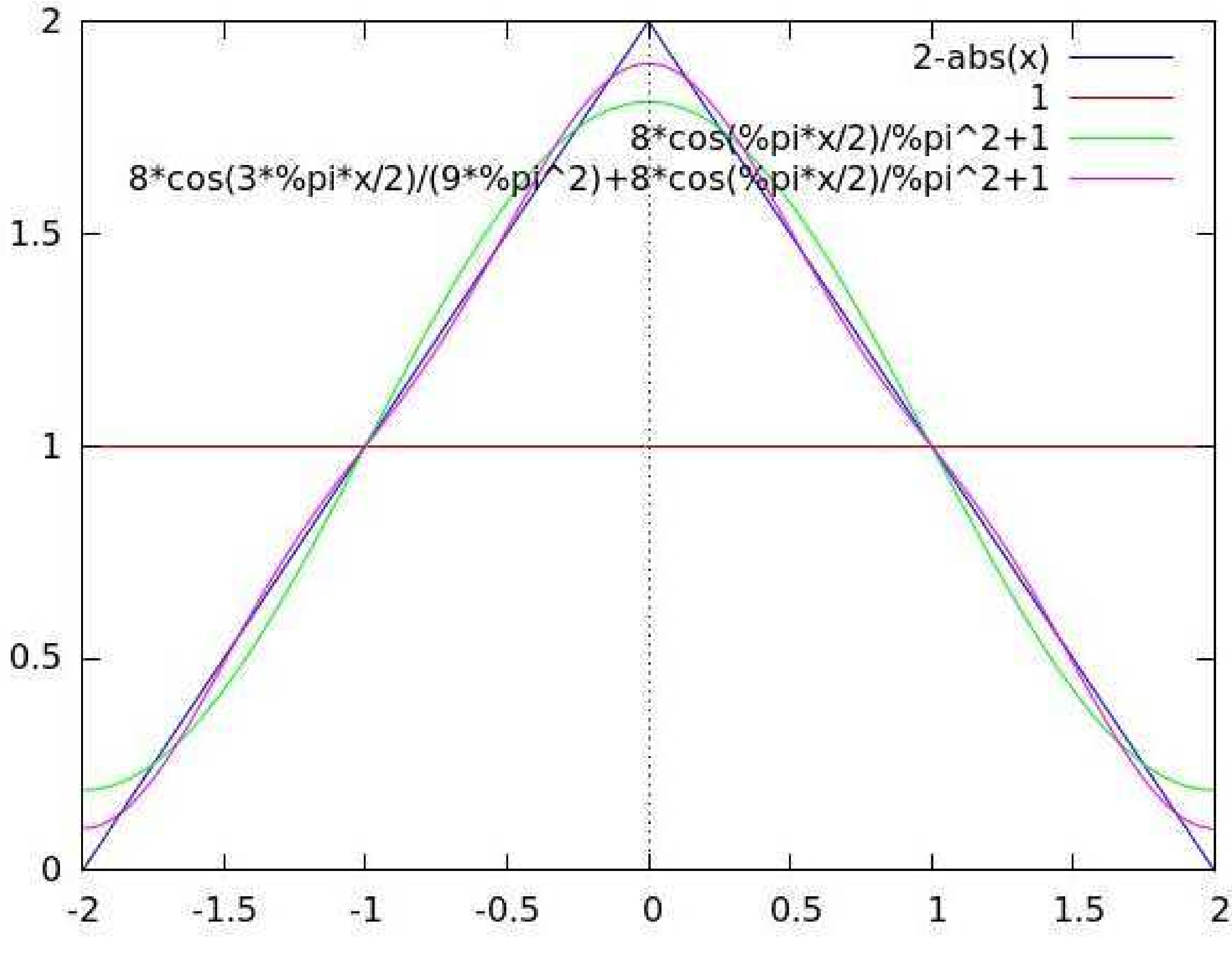
(%i35)
```
declare(n,integer)$
an:ratsimp(integrate(f*cos(%pi*n*x/2),x,0,2));
a0:(1/2)*integrate(f,x,0,2);
makelist(substitute(n=k,an),k,1,3);
a0+sum(substitute(n=k,an)*cos(k*%pi*x/2),k,1,3);
append([f],makelist(
 a0+sum(substitute(n=k,an*cos(n*%pi*x/2)),k,1,j),j,[0,1,3]))$
plot2d(%,[x,-2,2])$
```

(%o36) $-\frac{4\left(-1\right)^{n}-4}{\pi^{2}\,n^{2}}$

(%o37) 1

(%o38) $[\frac{8}{\pi^{2}},0,\frac{8}{9\,\pi^{2}}]$

(%o39) $\frac{8\cos\left(\frac{3\,\pi\,x}{2}\right)}{9\,\pi^{2}}+\frac{8\cos\left(\frac{\pi\,x}{2}\right)}{\pi^{2}}+1$


2-abs(x)
1
8*cos(%pi*x/2)/%pi^2+1
8*cos(3*%pi*x/2)/(9*%pi^2)+8*cos(%pi*x/2)/%pi^2+1
2
1.5
1
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

2

```
(%i25) u:sin(3*%pi*x/L)*cos(c*3*%pi*t/L);
       diff(diff(u,t),t)-c^2*diff(diff(u,x),x);
       substitute(t=0,u);
       substitute(t=0,diff(u,t));
```

(%o25) $\cos\left(\frac{3\pi c t}{L}\right)\sin\left(\frac{3\pi x}{L}\right)$

(%o26) 0

(%o27) $\sin\left(\frac{3\pi x}{L}\right)$

(%o28) 0

3

```
(%i32) u:23-(2/5^3)*sin(3*t)*r^3;
       diff(diff(u,r),r)+1/r*diff(u,r)+1/r^2*diff(diff(u,t),t);
       substitute(r=5,u);
```

(%o32) $23-\frac{2 r^3 \sin(3 t)}{125}$

(%o33) 0

(%o34) $23-2\sin(3 t)$