Midterm 2 MAT 3243.001 Fall 2005

① objective : cost $f = 2q_1^2 + q_1 q_2 + q_2^2 + 500$

constraint : $g = q_1 + q_2 - 200$

Let $\mathcal{L} = f - \lambda g = 2q_1^2 + q_1 q_2 + q_2^2 + 500 - \lambda(q_1 + q_2 - 200)$

$$\text{grad } \mathcal{L} = [\ 4q_1 + q_2 - \lambda,\ q_1 + 2q_2 - \lambda,\ -(q_1 + q_2 - 200)\ ]$$

sub

$$\text{grad } \mathcal{L} = 0 \Rightarrow 3q_1 - q_2 = 0 \Rightarrow q_2 = 3q_1$$

$$q_1 + 3q_1 = 200 \Rightarrow q_1 = 50, \quad q_2 = 150$$

∴ The 1st location should produce <u>50 axles</u> and the 2nd location <u>150 axles</u>

(2) Parametrize:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\left(\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2t \\ 1+t \\ t \end{bmatrix}, \quad 0 \le t \le 1$$

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} dt$$

$$\int \eta = \int_0^1 \left[-(1+t)\cdot 2 + (1+2t)\cdot 1 + t \cdot 1 \right] dt$$

$$= \int_0^1 (t-1)\, dt = \left[\frac{t^2}{2} - t \right]_0^1 = \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$$

$$d\eta = -dy\, dx + dx\, dy + \underbrace{dz\, dz}_{0} = \boxed{2\, dx\, dy}$$

i.e. curl $[-y, x, z] \neq 0$

Since $d\eta \neq 0$, the integral is path dependent.

(3) $\underline{X} = \begin{bmatrix} st \\ s+t \\ s-t \end{bmatrix}$, $D\underline{X} = \begin{bmatrix} t & s \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$

If $\begin{bmatrix} st \\ s+t \\ s-t \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ then $s = 2$, $t = -1$

Linear approximation @ $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$:

$$\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} s-2 \\ t+1 \end{bmatrix} = \boxed{\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}(s-2) + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}(t+1)}$$

$$= \begin{bmatrix} 2-s+2t \\ s+t \\ s-t \end{bmatrix}$$ } Not surprizingly ü

$$N = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$$

Eq: $\boxed{-2(x+2) + (y-1) - 3(z-3) = 0}$

i.e. $2x - y + 3z = 4$

(4) $F = \begin{bmatrix} x \\ y \\ z-1 \end{bmatrix}$

For the unit disc $d\bar{S} = \hat{k}\, r\, dr\, d\theta$

so $\displaystyle\int \bar{F} \cdot d\bar{S} = \int_{-\pi}^{\pi}\int_{0}^{1} \underbrace{(-1)}_{z-1 \text{ but } z=0} r\, dr\, d\theta = \boxed{-\pi}$

Let Ω = the top half of the unit ball 

$\operatorname{div} F = 3$ Divergence theorem:

$$\iiint_{\Omega} \operatorname{div} F \, dV = \oiint_{\partial\Omega} \bar{F} \cdot d\bar{S}$$

$$3 \cdot \frac{1}{2} \cdot \frac{4}{3}\pi = 2\pi \qquad \qquad \underbrace{\iint}_{\text{hemisphere}} + \underbrace{\iint}_{\substack{\text{disc with} \\ \text{downward normal}}}$$

disc with downward normal $= \pi$

$$\therefore \iint_{\text{hemisphere}} = 2\pi - \pi = \boxed{\pi}$$