Mid 2 F '03

Note Title 11/25/2008

① $x^4 - y^3 = 0 \qquad [-1,1] \to [1,1]$

Let $\bar{X}(t) = \begin{bmatrix} t^3 \\ t^4 \end{bmatrix} \quad -1 \le t \le 1$

$\leftarrow x$

$d\underline{X} = \begin{bmatrix} 3t^2 \\ 4t^3 \end{bmatrix} dt$

dy

$$\int x\,dy = \int_{-1}^{1} t^3 \cdot 4t^3\,dt = 4\int_{-1}^{1} t^6\,dt = \frac{8}{7}$$

② $d\omega = (2x-3y)\,dx + (4y-3x)\,dy$

$\omega = x^2 - 3xy + h(y)$

$d\omega = (2x-3y)\,dx - 3x\,dy + h'(y)\,dy$

$\therefore \quad h'(y) = 4y \qquad h(y) = 2y^2 + C$

$\therefore \quad \underline{\omega = x^2 - 3xy + 2y^2 + C}$

④ $F = [x, 3y, 0] \qquad x^2 + y^2 = 4 \quad -1 \le z \le 1$

Parametrize $\quad \underline{X}(\theta, z) = \begin{bmatrix} 2\cos\theta \\ 2\sin\theta \\ z \end{bmatrix}$

$\perp$ to the cylinder

$d\underline{X} = \begin{bmatrix} -2\sin\theta\,d\theta \\ 2\cos\theta\,d\theta \\ dz \end{bmatrix} \qquad d\underline{S} = \begin{bmatrix} dy\,dz \\ dz\,dx \\ dx\,dy \end{bmatrix} = \begin{bmatrix} 2\cos\theta \\ 2\sin\theta \\ 0 \end{bmatrix} d\theta\,dz$

$$\int \bar{F}\cdot d\bar{S} = \int_{-1}^{1}\left[\int_{-\pi}^{\pi}\left[(2\cos\theta)^2 + 12(\sin\theta)^2\right]d\theta\right]dz$$

$$= \int_{-1}^{1}\left[\left(4\cdot\tfrac{1}{2} + 12\cdot\tfrac{1}{2}\right)2\pi\right]dz = 16\pi\cdot 2 = \boxed{32\pi}$$

(5) $F = [\, x + \cos(yz),\ e^{xz} + y,\ z - \sin(yx) \,]$

Flux through unit sphere

Key: unit sphere $= \partial$ (unit ball)

$$\underset{\text{unit sphere}}{\iint} \bar{F} \cdot d\bar{S} = \underset{\text{unit ball}}{\iiint} \operatorname{div} F \, dV$$

$\operatorname{div} F = 3$

$3 \cdot \text{vol (unit Ball)}$

$= 3 \cdot \frac{4}{3}\pi = \boxed{4\pi}$

3. Find an equation and a parametric formula for the plane tangent to the surface $[e^s, t^2e^{2s}, 2e^{-s}+t]$ at $[1,4,0]$.

$e^s = 1 \Rightarrow s=0 \Rightarrow 2e^{-s}+t = 2+t \quad \therefore [s_0,t_0]=[0,-2]$

$$\Phi_s = [\,e^s,\ 2t^2e^{2s},\ -2e^{-s}\,]$$

$$\Phi_t = [\,0,\ 2te^{2s},\ 1\,]$$

$T_s = \Phi_s[s_0,t_0] = [1, 8, -2]$ $\quad [a,b,c] \perp \text{plane} \rightarrow$

$T_t = \Phi_t[s_0,t_0] = [0,-4,1]$ $\quad a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$T_s \times T_t = [0,-1,-4]$

$-(y-4)-4z=0$ or $\boxed{y+4z=4}$

Linearization: $\Phi[s_0,t_0] + D\Phi[s_0,t_0]\begin{bmatrix} s-s_0 \\ t-t_0 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 8 & -4 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} s \\ t+2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+s \\ 4+8s-4t-8 \\ -2s+t+2 \end{bmatrix} = \boxed{\begin{bmatrix} 1+s \\ -4+8s-4t \\ 2-2s+t \end{bmatrix}}$$

6. Let $\omega = xy$ and $\eta = y\,dx + x\,dz$. Find and simplify $d\omega \wedge \eta$ and $d\omega \wedge d\eta$.

$$d\omega = dx \cdot y + x \cdot dy = y\,dx + x\,dy$$

$$d\omega \wedge \eta = (y\,dx + x\,dy) \wedge (y\,dx + x\,dz)$$

$$= yx\,dx\,dz + xy\,dy\,dx + x^2\,dy\,dz$$

$$= x^2\,dy\,dz - yx\,dz\,dx - xy\,dx\,dy$$

$$d\eta = dy\,dx + dx\,dz = -dz\,dx - dx\,dy$$

$$d\omega \wedge d\eta = (y\,dx + x\,dy) \wedge (-dz\,dx - dx\,dy)$$

$$= -x\,dy\,dz\,dx = -x\,dx\,dy\,dz$$