① $6x \equiv 16 \mod 28$

$\gcd(6, 28) = 2$

$3x \equiv 8 \mod 14$

$3^{-1} = 5$

$x \equiv 40 \mod 14 \equiv 12 \mod 14$

$x = \ldots, -2, 12, 26, 40, \ldots$

② S'pose $a a' = 1 \quad a a'' = 1$

then $a a' a'' = a''$ (multiply 1st by a'')

$a a'' a' = a''$

$1 \cdot a' = a''$

$a' = a''$ ☺

③ Suppose $\phi : R \to S$ is a ring hom

$1 = \phi(1) = \phi(x x^{-1}) = \phi(x)\phi(x^{-1})$

$\therefore \phi(x^{-1}) = \phi(x)^{-1}$

(4) $\phi : R \to R$ $\quad \phi(x) = ax$

Hom: $\phi(x+y) = a(x+y) = ax + ay = \phi(x) + \phi(y)$

Suppose ϕ is 1-1.

If $a = 0$, then $\forall x$ $\phi(x) = 0$ ($\ker \phi = R$)

If a is a zero div. $\exists x \neq 0$, $ax = 0$,

so $x \in \ker \phi$, so $\ker \phi$ is nontrivial

Now suppose ϕ is not 1-1, then $\ker \phi$ is nontrivial, so $\exists x \neq 0$, $\phi(x) = 0$,

i.e. $ax = 0$. $\therefore$ a is zero or zero div.