1. $z^5 e^{z^2} = z^5 \sum_{n=0}^{\infty} \frac{1}{n!}(z^2)^n = \sum_{n=0}^{\infty} \frac{1}{n!} z^{2n+5} = z^5 + z^7 + \frac{1}{2}z^9 + ...$

   Since the series for e^z converges everywhere, the radius of convergence is ∞.

2. $$\frac{1}{z^2-4} = \frac{1}{(z-2)(z+2)} = \frac{1}{(z+2)(z+2-4)} = -\frac{1}{4(z+2)(1-\frac{z+2}{4})}$$
   $$= -\frac{1}{4(z+2)} \sum_{n=0}^{\infty} \left(\frac{z+2}{4}\right)^n = -\sum_{n=0}^{\infty} \frac{1}{4^{n+1}}(z+2)^{n-1}$$
   $$= -\frac{1}{4} \cdot \frac{1}{z+2} - \frac{1}{16} - \frac{1}{64}(z+2) - ...$$

   Since the geometric series converges on the unit disc and $\left|\frac{z+2}{4}\right| < 1 \Leftrightarrow |z+2| < 4$, the annulus of convergence is $\{z : 0 < |z+2| < 4\}$.

3. $\sin z = \frac{e^{iz} - e^{-iz}}{2i} = 0 \Leftrightarrow e^{iz} = e^{-iz} \Leftrightarrow e^{2iz} = 1 \Leftrightarrow 2iz = 2k\pi i, k \in \mathbf{Z} \Leftrightarrow z = k\pi,\ k \in \mathbf{Z}.$

   Expand in Taylor series at $k\pi$

   $$\sin z = \sin(z - k\pi + k\pi) = \frac{e^{i(z-k\pi+k\pi)} - e^{-i(z-k\pi+k\pi)}}{2i} = \frac{e^{i(z-k\pi)}e^{ik\pi} - e^{-i(z-k\pi)}e^{-ik\pi}}{2i}$$
   $$= \frac{e^{i(z-k\pi)}(-1)^k - e^{-i(z-k\pi)}(-1)^k}{2i} = (-1)^k \sin(z-k\pi) = (-1)^k(z-k\pi) + ...$$

   By long division (leading terms only) $\frac{1}{\sin z} = \frac{(-1)^k}{z-k\pi} + ...$, so

   $$\frac{z}{\sin z} = \frac{(-1)^k z}{z-k\pi} + ... = \frac{(-1)^k(k\pi + z - k\pi)}{z-k\pi} + ... = \frac{(-1)^k k\pi}{z-k\pi} + (-1)^k + ...$$

   Therefore, we have simple poles at $k\pi$ for $k \neq 0$ and a removable singularity at 0.

4. Let $\varepsilon > 0$. Then there exists n^* such that for $n \geq n^*$ we have $|f_n(z) - z| < \varepsilon$ for all z. By Liouville's theorem entire bounded functions are constant, so $f_n(z) - z = c$ for some c, so $f_n(z) = z + c$.