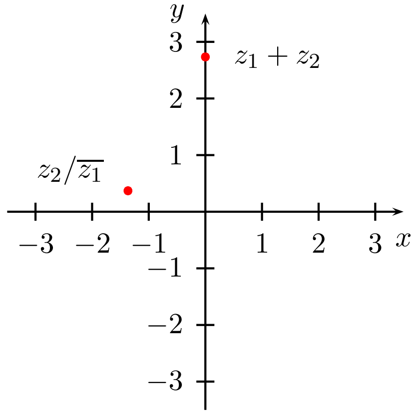
1.



$z_1 = 1 + i = \sqrt{2}e^{i\pi/4}$,
$z_2 = 2e^{i\,2\pi/3} = -1 + i\sqrt{3}$

$z_1 + z_2 = i(1 + \sqrt{3}) \approx i\,2.732$

$$z_2/\overline{z_1} = \frac{-1 + i\sqrt{3}}{1 - i} = \frac{-1 + i\sqrt{3}}{1 - i} \cdot \frac{1 + i}{1 + i}$$
$$= \frac{(-1 + i\sqrt{3})(1 + i)}{2} = -\frac{1 + \sqrt{3}}{2} + i\frac{\sqrt{3} - 1}{2}$$
$$\approx -1.366 + i\,0.366$$

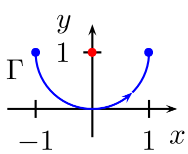
2. $f(z) = 3e^{i\pi/3}z + 3i$

3. If $f(x + iy) = x^2 + y^2 + i\,2xy$, then $\Re\,[f] = u = x^2 + y^2$ and $\Im\,[f] = v = 2xy$, so

$$\begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} = \begin{bmatrix} 2x & 2y \\ 2y & 2x \end{bmatrix}$$

The first Cauchy-Riemann equation $u_x = v_y$ is always satisfied, but the second $u_y = -v_x$ only holds when $y = 0$. Thus, the set where f is complex differentiable is the x axis.

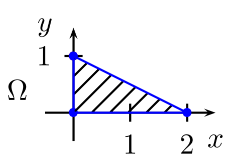
4.



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos t \\ \sin t + 1 \end{bmatrix}, \ -\pi \le t \le 0, \text{ so } \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} dt,$$

$$\int_\Gamma y\,dx - x\,dy = \int_{-\pi}^{0} (\sin t + 1)(-\sin t)\,dt - \cos t \cos t\,dt = \int_{-\pi}^{0} [-(\sin t)^2 - \sin t - (\cos t)^2]\,dt$$
$$= \int_{-\pi}^{0} (-1 - \sin t)\,dt = [-t + \cos t]_{-\pi}^{0} = 2 - \pi$$

5.



$$\iint_\Omega xy^2\,dx\,dy = \int_0^2 \left[\int_0^{-\frac{x}{2}+1} xy^2\,dy\right] dx = \int_0^2 \left[\frac{xy^3}{3}\right]_0^{-\frac{x}{2}+1} dx$$

$$= \int_0^2 \frac{x(-\frac{x}{2}+1)^3}{3}\,dx = \int_0^2 \frac{x}{3}\left[-\frac{x^3}{8} + \frac{3x^2}{4} - \frac{3x}{2} + 1\right] dx = \int_0^2 \left[-\frac{x^4}{24} + \frac{x^3}{4} - \frac{x^2}{2} + \frac{x}{3}\right] dx$$
$$= \left[-\frac{x^5}{120} + \frac{x^4}{16} - \frac{x^3}{6} + \frac{x^2}{6}\right]_0^2 = -\frac{4}{15} + 1 - \frac{4}{3} + \frac{2}{3} = \frac{1}{15}$$