(1) (i) Reflexive. Let $x \in X$, then $f(x) = f(x)$,
so $x \sim x$.

(ii) Symmetric: Suppose $x, x' \in X$, $x \sim x'$.
Then $f(x) = f(x')$, so $f(x') = f(x)$, so $x' \sim x$.

(iii) Transitive: Suppose $x, x', x'' \in X$,
$x \sim x'$, $x' \sim x''$. Then $f(x) = f(x')$, $f(x') = f(x'')$
so $f(x) = f(x'')$, so $x \sim x''$. ☺

a) If f is injective, $\forall x \in X$ $[x] = \{x\}$.
Pf if $x \sim x'$, then $f(x) = f(x')$, so
since f is inj., $x = x'$. ☺

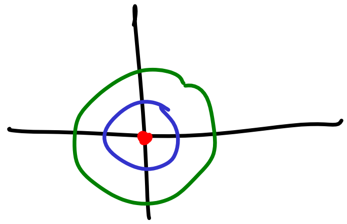
b) If f is const, $\forall x \in X$ $[x] = X$.
Pf If $x' \in X$, then $f(x) = f(x')$ = const.
so $x \sim x'$. ☺

c) $x \sim x' \in \mathbb{Z}$ $\iff$ $x \equiv x' \bmod 2$

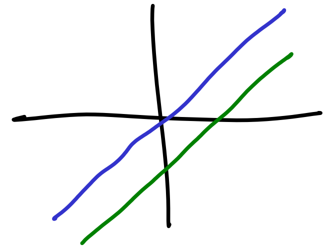
$\therefore$ 2 eq. classes: $\{\text{odds}\}$, $\{\text{evens}\}$

" $\{x \in \mathbb{Z} : \exists k \ \ x = 2k+1\}$ " $\{x \in \mathbb{Z} : \exists k \ \ x = 2k\}$

2 a)



b)



$x - y = 0 \to y = x$

$x - y = 1 \to y = x - 1$

3) $S = \{x \in \mathbb{Q} : \exists n \in \mathbb{N} \ \ x = \frac{1}{n}\} = \{1, \frac{1}{2}, \frac{1}{3}, \dots\}$

$\max S = 1 \quad \therefore \sup S = 1$.

pf $1 \in S$, If $\frac{1}{n} \in S$, then $1 \geq \frac{1}{n}$:)

$\nexists \min S$. pf Let $\frac{1}{n} \in S$, then $\frac{1}{n} > \frac{1}{n+1}$:)

$\inf S = 0$. pf $\forall n \quad 0 < \frac{1}{n}$, so

0 is a lower bd. for S.

If $x > 0$, since $x \in \mathbb{Q}$, $x = \frac{m}{n}$ for some $m, n \in \mathbb{N}$. Then since $m \geq 1$

$x \geq \frac{1}{n}$, so $x > \frac{1}{n+1}$, so

x is not a lower bound for S.

(4) 1. $\bigcup_{s\in S'} A_s$ is a ray to the left:

Let $y \in \bigcup_{s\in S'} A_s$, and $x<y$

$\exists\, t \in S'$ $y \in A_t$

Since A_t is a ray to the left, $x \in A_t$,

so $x \in \bigcup_{s\in S'} A_s$ ☺

2. $\bigcup_{s\in S'} A_s$ has no max:

Let $y \in \bigcup_{s\in S'} A_s$.

$\exists\, t \in S'$ $y \in A_t$

Since A_t has no max $\exists\, y' \in A_t$ $y<y'$,

But then $y' \in \bigcup_{s\in S'} A_s$ ☺

(5) 1. If $\mathbb{N}$ were bounded above in $\mathbb{R}$,

we would have $s = \sup \mathbb{N}$ (by completeness)

Then $s - \frac{1}{2}$ is not an upper bd for $\mathbb{N}$, so

$\exists n \in \mathbb{N}$ $\quad n > s - \frac{1}{2}$,

Then $n + 1 > s - \frac{1}{2} + 1 = s + \frac{1}{2} > s$ ☹

2. Since $\mathbb{N}$ is not bdd above,

$\exists n \in \mathbb{N}$ $\quad \frac{y}{x} < n$

Then $y < nx$ $\quad (x > 0)$ ☺