(1) $\begin{bmatrix} 2x+4y+6z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=6 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & 4 & 6 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 3 \\ 7 & 8 & 9 & 6 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 3 \\ 0 & -6 & -12 & 6 \end{array}\right]$$

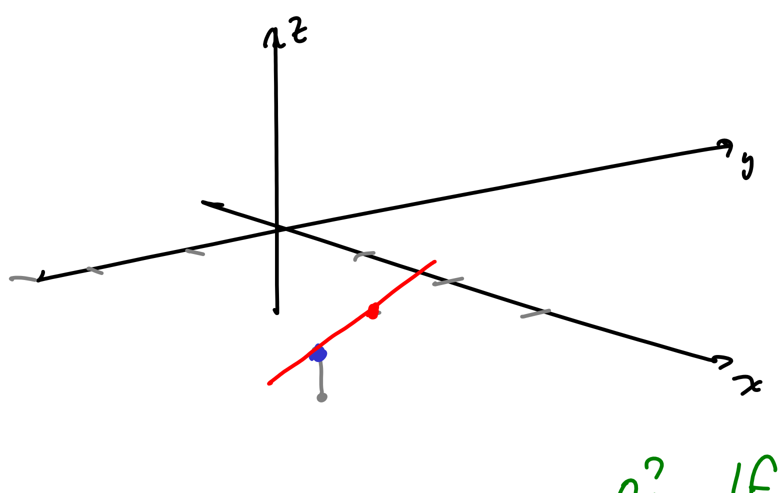
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & -6 & -12 & 6 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x-z=2 \\ y+2z=-1 \end{array}$$

↑ free

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z+2 \\ -2z-1 \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + z \underbrace{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}_{\text{direction}}$$

Since we have 1 free var., the solution is a line.

$z=0 \Rightarrow \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $z=1 \Rightarrow \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$



b) $\left[\begin{array}{ccc|c} 1 & 0 & -1 & * \\ 0 & 1 & 2 & * \\ 0 & 0 & 0 & * \end{array}\right]$ is this 0? If yes, there sol'n. If not, no solutions.

↑ free

② $T: \mathbb{R}^2 \to \mathbb{R}$ $\qquad T\bar{u} = a \qquad T\bar{v} = b$

$\bar{u}$ and $\bar{v}$ are not scalar mult. of one another.

a) $S = [\bar{u} \quad \bar{v}]$

$= \begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \end{bmatrix}$ (in coords ◡̈)

If $\bar{u} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then $\bar{u} = 0 \cdot \bar{v}$ ⌒̈

u_1 or u_2 is nonzero, if $u_1 = 0$, swap the rows.

$\therefore$ Without loss of generality assume $u_1 \neq 0$.

Gauss-Jordan: $\begin{bmatrix} 1 & v_1/u_1 \\ u_2 & v_2 \end{bmatrix} \to \begin{bmatrix} 1 & v_1/u_1 \\ 0 & v_2 - u_2 \frac{v_1}{u_1} \end{bmatrix}$

If $v_2 - u_2 \frac{v_1}{u_1} = 0$, $v_2 = u_2 \frac{v_1}{u_1}$ $\qquad \left(v_1 = \cancel{u_1} \frac{v_1}{\cancel{u_1}} \right)$

$\therefore \quad \bar{v} = \frac{v_1}{u_1} \bar{u}$ ⌒̈

$\therefore$ We have 2 pivots $\therefore$ S is invertible

$\therefore$ $\text{rref}(S) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b) Suppose A represents T (1×2)

$$A = [A\bar{e}_1 \quad A\bar{e}_2] = [T(e_1) \quad T(e_2)]$$

$$AS = [AS\bar{e}_1 \quad AS\bar{e}_2] = [T(S\bar{e}_1) \quad T(Se_2)]$$

$$= [T(\bar{u}) \quad T(\bar{v})] = [a \quad b] = B$$

:)

3. c) $AS = B \qquad ASS^{-1} = BS^{-1} \qquad A = BS^{-1}$

d) $S = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \qquad B = [1 \quad -2]$

$$S^{-1} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} \qquad \boxed{A = BS^{-1} = [4 \quad 7]}$$

4. a) Refl. Let $L =$ x-axis $\qquad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

b) proj $\qquad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

c) iso. dilation $\qquad \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix}$

d) rotation $\qquad$ Let $\theta = \pi$ $\qquad \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

e) horiz shear $\qquad \begin{bmatrix} 1 & 42 \\ 0 & 1 \end{bmatrix}$