MAT 2233.002 Mid 3 Sp '16

① $A^TA = \begin{bmatrix}0&1&1\\1&2&0\end{bmatrix}\begin{bmatrix}0&1\\1&2\\1&0\end{bmatrix} = \begin{bmatrix}2&2\\2&5\end{bmatrix}$, $A^Tb = \begin{bmatrix}0&1&1\\1&2&0\end{bmatrix}\begin{bmatrix}3\\-3\\3\end{bmatrix} = \begin{bmatrix}0\\-3\end{bmatrix}$

$\text{rref}\left[\begin{array}{cc|c}2&2&0\\2&5&-3\end{array}\right] = \left[\begin{array}{cc|c}1&0&1\\0&1&-1\end{array}\right]$, $\boxed{x^* = \begin{bmatrix}1\\-1\end{bmatrix}}$, $b - Ax^* = \begin{bmatrix}3\\-3\\3\end{bmatrix} - \begin{bmatrix}0&1\\1&2\\1&0\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}4\\-2\\2\end{bmatrix}$

$\begin{bmatrix}4&-2&2\end{bmatrix}\begin{bmatrix}0&1\\1&2\\1&0\end{bmatrix} = \begin{bmatrix}0&0\end{bmatrix}$, so $b - Ax^* \perp \text{image}(A)$ = span of cols of A

② Let $\mathcal{B} = [1, t, t^2, t^3]$, then $\mathcal{B}$ is a basis for P_3

$T(1) = 1$, $T(t) = t$, $T(t^2) = 2t + t^2$, $T(t^3) = 6t^2 + t^3$

$\therefore$ relative to $\mathcal{B}$, T is represented by $\begin{bmatrix}1&0&0&0\\0&1&2&0\\0&0&1&6\\0&0&0&1\end{bmatrix}$, so $\boxed{\det T = 1}$

③ a) $A = \begin{bmatrix}-7&3\\-18&8\end{bmatrix}$, $\det(A - \lambda I) = \det\begin{bmatrix}-7-\lambda & 3\\-18 & 8-\lambda\end{bmatrix}$

$= (-7-\lambda)(8-\lambda) + 54 = \lambda^2 - 8\lambda + 7\lambda - 56 + 54$

$= \lambda^2 - \lambda - 2$ $\boxed{\lambda = 2, -1}$

Eigenvectors

$A - 2I = \begin{bmatrix}-9&3\\-18&6\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}1&-\frac{1}{3}\\0&0\end{bmatrix}$ $x = \frac{1}{3}y$ Let $\bar{v}_1 = \begin{bmatrix}1\\3\end{bmatrix}$

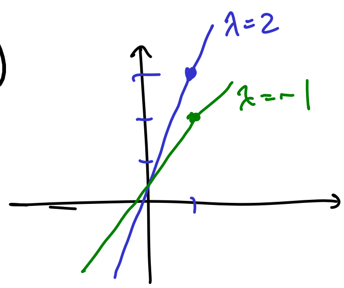
$A + I = \begin{bmatrix}-6&3\\-18&9\end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix}1&-\frac{1}{2}\\0&0\end{bmatrix}$ $x = \frac{1}{2}y$ Let $\bar{v}_2 = \begin{bmatrix}1\\2\end{bmatrix}$

b) $S = \begin{bmatrix}1&1\\3&2\end{bmatrix}$ $AS = \begin{bmatrix}2&-1\\6&-2\end{bmatrix}$ $S^{-1} = \begin{bmatrix}-2&1\\3&-1\end{bmatrix}$

$\uparrow$ $2\bar{v}_1$ $(-1)\bar{v}_2$

$S^{-1}AS = \begin{bmatrix}2&0\\0&-1\end{bmatrix}$

c)



dilation by 2 along the blue line

reflection along the green line

④ Since eigenvectors are lin. indep., they form a basis for $\mathbb{R}^n$.

Relative to this basis the matrix is diagonal $\begin{bmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_n\end{bmatrix}$, so $\boxed{\det = \lambda_1\lambda_2\cdots\lambda_n}$

Note: $\det(S^{-1}AS) = \det A$

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(%i1) load("d.mac")$
```

1

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(%i2) A:matrix([0,1],[1,2],[1,0]);
      b:transpose(matrix([3,-3,3]));
      addcol(transpose(A).A,transpose(A).b);
      rref(%)$
      xls:col(%,3);
      b-A.xls;
      transpose(%).A;
```

(%o2) $\begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$

(%o3) $\begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$

(%o4) $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & -3 \end{bmatrix}$

(%o6) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(%o7) $\begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$

(%o8) $\begin{bmatrix} 0 & 0 \end{bmatrix}$

2

```
(%i215) T(p):=t*diff(diff(p,t),t)+p$
        genmatrix(lambda([i,j],coeff(T(t^(j-1)),t,(i-1))),4,4);
        determinant(%);
```

(%o216) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(%o217) 1

3

```
(%i133) A:matrix([-7,3],[-18,8]);
        X:A-l*ident(2);
        determinant(%); ratsimp(%);
        ss:solve(%,l);
        substitute(ss[1],X);
        rref(%);
        substitute(ss[2],X);
        rref(%);
        ee:eigenvectors(A);
        S:transpose(matrix(ee[2][1][1],ee[2][2][1]));
        invert(S);
        %.A.S;
```

(%o133) $\begin{bmatrix} -7 & 3 \\ -18 & 8 \end{bmatrix}$

(%o134) $\begin{bmatrix} -l-7 & 3 \\ -18 & 8-l \end{bmatrix}$

(%o135) $(-l-7)(8-l)+54$

(%o136) l^2-l-2

(%o137) $[l=2, l=-1]$

(%o138) $\begin{bmatrix} -9 & 3 \\ -18 & 6 \end{bmatrix}$

(%o139) $\begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & 0 \end{bmatrix}$

(%o140) $\begin{bmatrix} -6 & 3 \\ -18 & 9 \end{bmatrix}$

(%o141) $\begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix}$

(%o142) $[[[2,-1],[1,1]],[[[1,3]],[[1,2]]]]$

(%o143) $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

(%o144) $\begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$

(%o145) $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$