

Mid 1 Fall '02

1. $A = \begin{bmatrix} 2 & -6 & -2 \\ -1 & 3 & -3 \end{bmatrix}$ $b = \begin{bmatrix} -2 \\ -7 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & -6 & -2 & -2 \\ -1 & 3 & -3 & -7 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -1 & -1 \\ 0 & 0 & -4 & -8 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

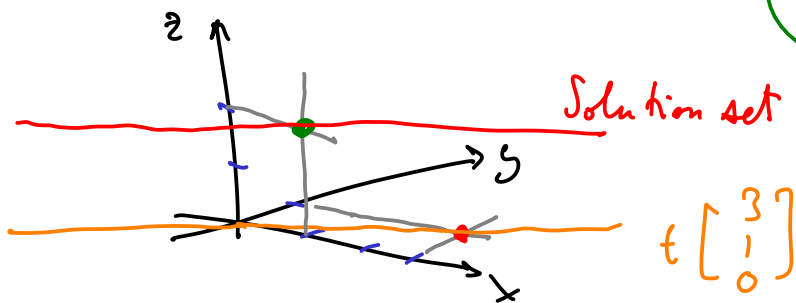
$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$x = 3y + 1$
 $z = 2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3y + 1 \\ y \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

(line along $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$)

passing through $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$



② T orth. projection to $x = -4y$

let $\bar{u} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, $|\bar{u}| = \sqrt{4^2 + 1^2} = \sqrt{17}$

$$T(e_1) = (e_1 \cdot \hat{u}) \hat{u}$$

$$= -\frac{4}{\sqrt{17}} \begin{bmatrix} -\frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{bmatrix}$$

$$\hat{u} = \frac{\bar{u}}{|\bar{u}|} = \begin{bmatrix} -\frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{bmatrix}$$

$$T(e_2) = (e_2 \cdot \hat{u}) \hat{u}$$

$$= \frac{1}{\sqrt{17}} \begin{bmatrix} -\frac{4}{\sqrt{17}} \\ \frac{1}{\sqrt{17}} \end{bmatrix}$$

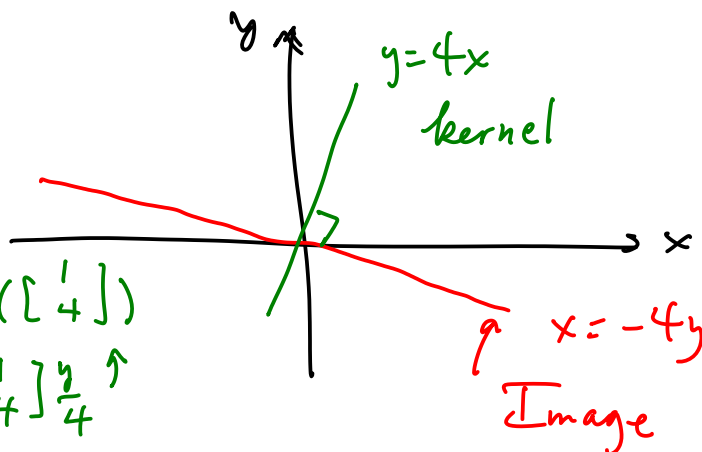
$$\begin{bmatrix} \frac{16}{17} & -\frac{4}{17} \\ -\frac{4}{17} & \frac{1}{17} \end{bmatrix}$$

↓ rref

$$\begin{bmatrix} 1 & -\frac{1}{4} \\ 0 & 0 \end{bmatrix}$$

$$\ker T = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x = \frac{1}{4}y \right\} = \text{span} \left(\begin{bmatrix} 1 \\ 4 \end{bmatrix} \right)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{4}y \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \frac{y}{4}$$



③ $A = 2 \times 1$ $\begin{bmatrix} * \\ * \end{bmatrix} \begin{bmatrix} * \end{bmatrix} = \begin{bmatrix} * \end{bmatrix}$

Aug $\begin{bmatrix} * & \vdots & * \\ * & \vdots & * \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & \vdots & * \\ 0 & \vdots & * \end{bmatrix}$

$A = 1 \times 2$

if 0, unique sol.
if $\neq 0$, no solutions

$$\begin{bmatrix} * & * \end{bmatrix} \begin{bmatrix} * \\ * \end{bmatrix} = \begin{bmatrix} * \end{bmatrix}$$

Aug $\begin{bmatrix} * & * & \vdots & * \end{bmatrix}$

∞ -many sol.

rref: $\begin{bmatrix} 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \end{bmatrix}$

$$\textcircled{4} \quad \begin{bmatrix} 2 \\ -7 \end{bmatrix} a + \begin{bmatrix} -1 \\ 3 \end{bmatrix} b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -7 \end{bmatrix} c + \begin{bmatrix} -1 \\ 3 \end{bmatrix} d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

stack $\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & -1 & | & 1 & 0 \\ -7 & 3 & | & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{7}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -7 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 1 & -7 & -2 \end{bmatrix}$$

$$-3 \begin{bmatrix} 2 \\ -7 \end{bmatrix} - 7 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{☺}$$

$$-1 \begin{bmatrix} 2 \\ -7 \end{bmatrix} - 2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{☺}$$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = -3 T\left(\begin{bmatrix} 2 \\ -7 \end{bmatrix}\right) - 7 T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right)$$

$$\begin{matrix} \underbrace{\phantom{-3 T\left(\begin{bmatrix} 2 \\ -7 \end{bmatrix}\right)}}_{\begin{bmatrix} 6 \\ -12 \\ -7 \end{bmatrix}} & \underbrace{\phantom{-7 T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right)}}_{\begin{bmatrix} -3 \\ 5 \\ 3 \end{bmatrix}} \end{matrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

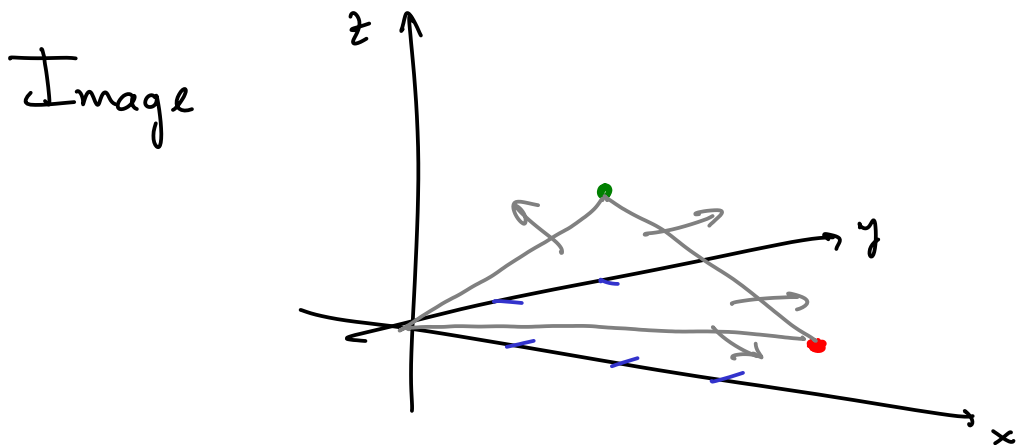
$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = - T\left(\begin{bmatrix} 2 \\ -7 \end{bmatrix}\right) - 2 T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{matrix} \underbrace{\phantom{- T\left(\begin{bmatrix} 2 \\ -7 \end{bmatrix}\right)}}_{\begin{bmatrix} 6 \\ -12 \\ -7 \end{bmatrix}} & \underbrace{\phantom{-2 T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right)}}_{\begin{bmatrix} -3 \\ 5 \\ 3 \end{bmatrix}} \end{matrix}$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

rank-nullity $2 = \underbrace{\dim(\ker(T))}_0 + \underbrace{\text{rk}(T)}_2$



$$\text{Image}(T) = \text{span}\left(\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\right) \text{ (plane)}$$

$$\ker(T) = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \text{ (point :)}\right.$$

T is 1-1, but not onto.