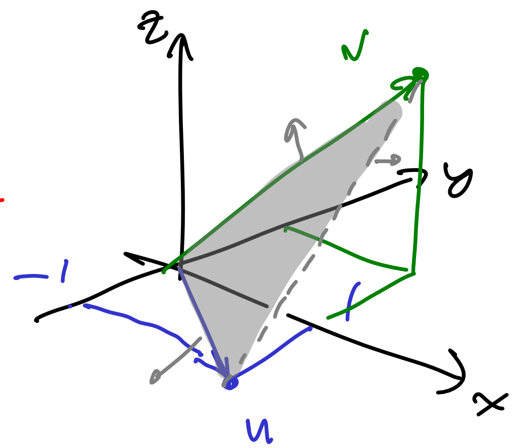
① $u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$n = u \times v = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$

$\boxed{-x - y + 3z = 0}$



② $|n| = \sqrt{1+1+9} = \sqrt{11}$ $\boxed{A = \frac{1}{2}\sqrt{11} = 3.3}$

$|u| = \sqrt{2} = 1.4$ $|v| = \sqrt{4+1+1} = \sqrt{6} = 2.4$ $|v-u| = \left| \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right| = \sqrt{6} = 2.4$

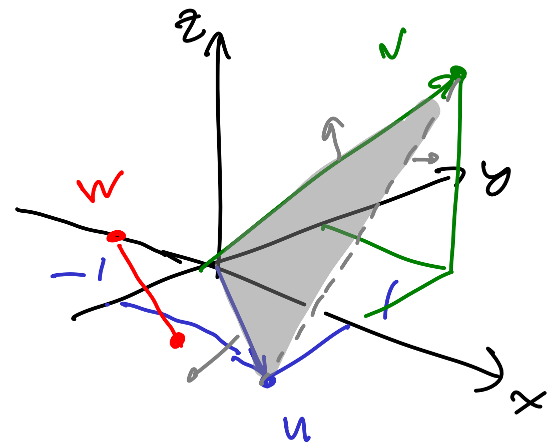
$\arccos\left(\frac{u \cdot v}{|u||v|}\right) = \arccos\left(\frac{1}{\sqrt{2}\sqrt{6}}\right) = \underline{1.3 = 73.2^\circ}$

$\arccos\left(\frac{u \cdot (u-v)}{|u||u-v|}\right) = \arccos\left(\frac{1}{\sqrt{2}\sqrt{6}}\right) = \underline{}$

$\arccos\left(\frac{v \cdot (v-u)}{|v||v-u|}\right) = \arccos\left(\frac{5}{6}\right) = \underline{0.6 = 33.6^\circ}$

Check: $2 \cdot 73.2 + 33.6$
$= 146.4 + 33.6$
$= 180^\circ$

③



$$r(t) = w + tn$$

$$= \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1-t \\ -t \\ 3t \end{bmatrix} \begin{matrix} \leftarrow x \\ \leftarrow y \\ \leftarrow z \end{matrix}$$

$$-x - y + 3z = 0$$

$$-(-1-t) - (-t) + 3 \cdot 3t = 0$$

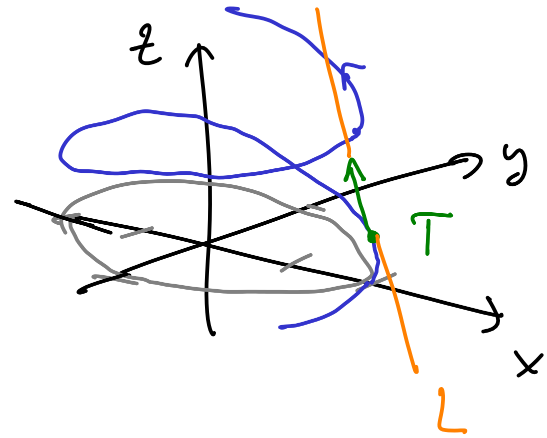
$$11t + 1 = 0 \qquad \boxed{t = -\frac{1}{11}}$$

$$r\left(-\frac{1}{11}\right) = \begin{bmatrix} -1+\frac{1}{11} \\ \frac{1}{11} \\ \frac{3}{11} \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -10 \\ 1 \\ 3 \end{bmatrix}$$

$$r\left(-\frac{1}{11}\right) - w = \frac{1}{11} \begin{bmatrix} -10 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{11} \\ \frac{1}{11} \\ \frac{3}{11} \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\left| r\left(-\frac{1}{11}\right) - w \right| = \frac{1}{11}\sqrt{1+1+9} = \boxed{\frac{1}{\sqrt{11}} = 0{,}3}$$

④ $r(t) = \begin{bmatrix} 2\cos t \\ \sin t \\ \frac{1}{2}t \end{bmatrix}$



$u = \begin{bmatrix} \sqrt{2} \\ \frac{1}{\sqrt{2}} \\ \frac{\pi}{8} \end{bmatrix}$ $\quad v = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$\frac{1}{2}t = 0$

$\boxed{t = 0}$

$2\cos(0) = 2$ ✓

$\sin(0) = 0$ ✓

$\frac{1}{2}t = \frac{\pi}{8} \Rightarrow \boxed{t = \frac{\pi}{4}}$

Check: $2\cos\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$ ✓

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ ✓

b) $r'(t) = \begin{bmatrix} -2\sin t \\ \cos t \\ 1/2 \end{bmatrix}$ $\quad r'\left(\frac{\pi}{4}\right) = \begin{bmatrix} -\sqrt{2} \\ 1/\sqrt{2} \\ 1/2 \end{bmatrix}$

$$\left|r'\left(\frac{\pi}{4}\right)\right| = \sqrt{2 + \frac{1}{2} + \frac{1}{4}} = \frac{1}{2}\sqrt{8+2+1} = \frac{\sqrt{11}}{2}$$

$$\hat{T} = \frac{r'(\pi/4)}{\left|r'\left(\frac{\pi}{4}\right)\right|} = \frac{2}{\sqrt{11}} \begin{bmatrix} -\sqrt{2} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -0.85 \\ 0.43 \\ 0.30 \end{bmatrix}$$

$$u + s\, r'\left(\frac{\pi}{4}\right) = \begin{bmatrix} \sqrt{2} \\ \frac{1}{\sqrt{2}} \\ \frac{\pi}{8} \end{bmatrix} + s \begin{bmatrix} -\sqrt{2} \\ 1/\sqrt{2} \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1.4 - 0.85s \\ 0.7 + 0.4s \\ 0.4 + 0.3s \end{bmatrix}$$

c)

$$\int |dr| = \int_0^{\pi/4} |r'|\,dt \qquad r'(t) = \begin{bmatrix} -2\sin t \\ \cos t \\ 1/2 \end{bmatrix}$$

$$= \int_0^{\pi/4} \sqrt{4(\sin t)^2 + (\cos t)^2 + \frac{1}{4}}\ dt$$

$$= \int_0^{\pi/4} \sqrt{3(\sin t)^2 + \frac{5}{4}}\ dt \approx 1$$

(5)

$$p_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \qquad p_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$r(t) = p_1(1-t) + p_2 t = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}(1-t) + \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} t$$

$$= \begin{bmatrix} 1+t \\ 1-2t \\ 2-2t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}}_{p_1} + t \underbrace{\begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}}_{p_2 - p_1} \qquad 0 \le t \le 1$$

b)

$$\int_{S'} \bar{F} \cdot d\bar{r} = ? \qquad dr = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix} dt$$

$$F = [xy,\ y,\ -yz] = [(1+t)(1-2t),\ 1-2t,\ -(1-2t)(2-2t)]$$

$$= [1 - t - 2t^2,\ 1-2t,\ -2+6t-4t^2]$$

$$F \cdot dr = 1 - t - 2t^2 + (1-2t)(-2) + (-2+6t-4t^2)(-2)$$

$$= 3 - 9t + 6t^2$$

$$\int F \cdot dr = \int_0^1 (3 - 9t + 6t^2)\,dt = 3 - \frac{9}{2} + \frac{6}{3} = \boxed{\frac{1}{2}}$$

⑥ a) $\omega = [2x+y]\,dx + [z\cos(yz)+x]\,dy + y\cos(yz)\,dz$

a) $d\omega = [\cancel{2dx} + dy]dx + [dz\cdot\cos(yz) + z(-\sin(yz))\cdot$

$\cdot(\cancel{dy\cdot z} + y\cdot dz) + dx]dy + [dy\cdot\cos(yz) + y(-\sin(y,z))\cdot$

$\cdot(dy\cdot z + \cancel{y\cdot dz})]\,dz$

$= dy\,dz\,[-\cancel{\cos(yz)} + yz\cancel{\sin(yz)} + \cancel{\cos(yz)} - zy\cancel{\sin(yz)}]$

$+ dz\,dx\cdot 0$

$+ dx\,dy\,[-1+1] = 0$

b) $\eta_x = 2x+y \qquad \eta = x^2 + yx + k(y,z)$

$\eta_y = x + k_y = z\cos(yz) + x$, so $k_y = z\cos(yz)$

$k = \underbrace{z\sin(yz)\frac{1}{z}}_{\sin(yz)} + h(z)$, $\eta = x^2 + yx + \sin(yz) + h(z)$

$\eta_z = \cos(yz)\cdot y + h' = y\cos(yz)$, so $h' = 0$, so $h = c$

$$\boxed{\eta = x^2 + yx + \sin(yz) + c}$$

$$\int_{\begin{bmatrix}0\\0\\0\end{bmatrix}}^{\begin{bmatrix}1\\2\\3\end{bmatrix}} \omega = \eta\Big|_{\begin{bmatrix}0\\0\\0\end{bmatrix}}^{\begin{bmatrix}1\\2\\3\end{bmatrix}} = 1 + 2 + \sin(6) - 0 = \boxed{3 + \sin(6) \approx 2.72}$$