1. $s = \begin{bmatrix} x \\ y \end{bmatrix} = a \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$, $-\pi < t \le \pi$, $ds = a \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} dt$

$F = \begin{bmatrix} -y \\ 2x \end{bmatrix} = a \begin{bmatrix} -\sin t \\ 2\cos t \end{bmatrix}$, $\int F \cdot ds = \int_{-\pi}^{\pi} a^2 [\underbrace{(\sin t)^2 + 2(\cos t)^2}_{3/2 \text{ on average}}] dt = \boxed{3\pi a^2}$

2-d curl of F = 3, integrate: $3\int dA = \boxed{3\pi a^2}$

$G = \begin{bmatrix} -x \\ 2y \end{bmatrix} = a \begin{bmatrix} -\cos t \\ 2\sin t \end{bmatrix}$, $dn = a \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} dt$

$\int G \cdot dn = a \int_{-\pi}^{\pi} [\underbrace{-(\cos t)^2 + 2(\sin t)^2}_{1/2 \text{ on average}}] dt = \boxed{\pi a^2}$

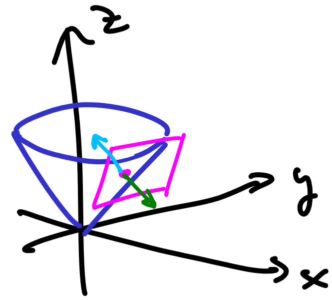
div G = 1, integrate: $\boxed{\pi a^2}$

2. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r\cos t \\ r\sin t \\ r \end{bmatrix}$, $0 \le r \le h$, $-\pi < t \le \pi$

$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} \cos t \, dr - r \sin t \, dt \\ \sin t \, dr + r\cos t \, dt \\ dr \end{bmatrix}$

$dS = \begin{bmatrix} dy \, dz \\ dz \, dx \\ dx \, dy \end{bmatrix} = \begin{bmatrix} -r\cos t \\ -r\sin t \\ r \end{bmatrix} dr \, dt$, $|dS| = \sqrt{2} \, r \, dr \, dt$

$\int |dS| = \sqrt{2} \int_{-\pi}^{\pi} \underbrace{\int_0^h r \, dr}_{h^2/2} \, dt = \boxed{\sqrt{2}\pi h^2}$



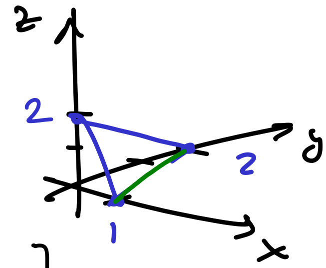
3. $d(x^2 + y^2 - z^2) = 2x\,dx + 2y\,dy - 2z\,dz$

Eval at pt.: $2dx + 2dy - 2^{3/2} dz$

Tangent plane: $2(x-1) + 2(y-1) - 2^{3/2}(z - \sqrt{2}) = 0$

Simplify: $\boxed{x + y - \sqrt{2} z = 0}$ unit normals: $\boxed{\pm \begin{bmatrix} 1/2 \\ 1/2 \\ -1/\sqrt{2} \end{bmatrix}}$

4. $\Omega: \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 2-2x-y \end{bmatrix} \quad \begin{matrix} 0 \le y \le 2-2x \\ 0 \le x \le 1 \end{matrix}$



$z=0 \Rightarrow y = 2-2x$

$F = \begin{bmatrix} xz \\ xy \\ 3xz \end{bmatrix} \qquad \operatorname{curl} F = \begin{bmatrix} 0 \\ x-3z \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 7x+3y-6 \\ y \end{bmatrix}$

$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} dx \\ dy \\ -2dx-dy \end{bmatrix} \qquad dS = \begin{bmatrix} dydz \\ dzdx \\ dxdy \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} dxdy$

$$\int \operatorname{curl} F \cdot dS = \int_0^1 \left[\int_0^{2-2x} (7x+4y-6)\, dy \right] dx = \int_0^1 \left[7xy + 2y^2 - 6y \right]_0^{2-2x} dx =$$

$$= \int_0^1 \underbrace{\left(7x(2-2x) + 2\underbrace{(2-2x)^2}_{4-8x+4x^2} - 6(2-2x) \right)}_{-6x^2+10x-4} dx = \left[-2x^3+5x^2-4x \right]_0^1 = \boxed{-1}$$

5. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a\cos t \\ a\sin t \\ z \end{bmatrix}, \quad \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} -a\sin t\, dt \\ a\cos t\, dt \\ dz \end{bmatrix}, \quad dS = \begin{bmatrix} dydz \\ dzdx \\ dxdy \end{bmatrix} = \underbrace{\begin{bmatrix} a\cos t \\ a\sin t \\ 0 \end{bmatrix}}_{\perp \text{ cyl. ☺}} dt\, dz$

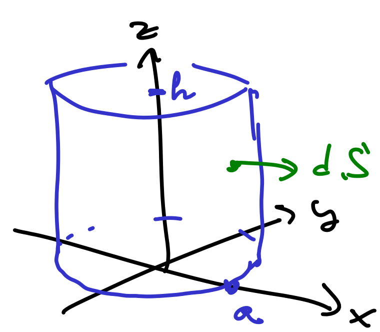
$G = \begin{bmatrix} -x \\ 2y \\ 0 \end{bmatrix} = \begin{bmatrix} -a\cos t \\ 2a\sin t \\ 0 \end{bmatrix}$

$$\int G \cdot dS = \int_0^h \int_{-\pi}^{\pi} \left(\underbrace{-a^2(\cos t)^2 + 2a^2(\sin t)^2}_{\frac{1}{2} \text{ on average}} \right) dt\, dz = \boxed{\pi a^2 h}$$

Since G is tangent to top and bottom, flux of G is zero there.

$\operatorname{div} G = 1$, integrate:

$$\int \operatorname{div} G\, dV = \int dV = \text{volume} = \boxed{\pi a^2 h}$$



6. $f = x^2+y^2-y+2, \quad df = 2x\,dx + (2y-1)\,dy$

$df = 0 \Rightarrow \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$ is the only crit. pt.

$Hf = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\quad \det Hf = 2 > 0 \Rightarrow$ min or max

$f_{xx} = 2 > 0 \Rightarrow$ min

7. Boundary: $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{3}{2}\begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$, plug into f:

$\frac{9}{4}(\cos t)^2 + \frac{9}{4}(\sin t)^2 - \frac{3}{2}\sin t + 2 = \frac{1}{4}(-6\sin t + 17)$

differentiate: $-\frac{3}{2}\cos t$. $\quad \cos t = 0 \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 3/2 \end{bmatrix}$

	$[x,y]$	$f(x,y)$	
interior	$[0, 1/2]$	$7/4$	← min
	$[0, 3/2]$	$11/4$	
	$[0, -3/2]$	$23/4$	← max

eval at $[1,2]$ $f \to e^2$

8. $f = e^{xy}$ $\quad Df = [ye^{xy}, xe^{xy}] \to [2e^2, e^2]$

$Hf = \begin{bmatrix} y^2e^{xy} & xye^{xy}+e^{xy} \\ xye^{xy}+e^{xy} & x^2e^{xy} \end{bmatrix} \to \begin{bmatrix} 4e^2 & 3e^2 \\ 3e^2 & e^2 \end{bmatrix}$

Linear: $L = f(1,2) + Df\begin{bmatrix} x-1 \\ y-2 \end{bmatrix} = e^2 + 2e^2(x-1) + e^2(y-2)$

$= \boxed{e^2(2x+y-3)}$

Quadratic: $Q = L + [x-1, y-2]\, Hf \begin{bmatrix} x-1 \\ y-2 \end{bmatrix} =$

$= L + 4e^2(x-1)^2 + 6e^2(x-1)(y-2) + e^2(y-2)^2$

$= \boxed{e^2(4x^2+y^2+6xy-18x-9y+17)}$

	f	L	Q
$[0.8, 1.8]$	2.956	6.207	4.221
$[0.8, 2.2]$	5.911	5.616	5.812
$[1.2, 1.8]$	8.867	8.571	8.671
$[1.2, 2.2]$	11.822	15.074	14.013