1. Find a parametrization for the line of intersection of the planes $x + 2y + 3z = 6$ and $x - y = 0$. Sketch.

Method 1

$$x + 2y + 3z = 6$$
$$x - y = 0 \Rightarrow x = y$$

so $3y + 3z = 6$

so $y + z = 2 \quad \therefore \quad y = 2 - z$

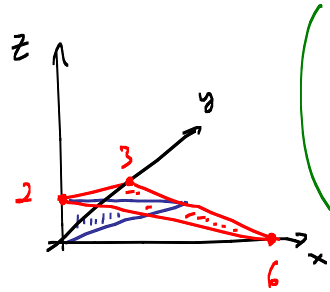
Let $z = t$ $\quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2-t \\ 2-t \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

Method 2 Pick 2 pts of intersection (by inspection)

say $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

Direction vector: $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

so $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ $\left(\text{or } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}(1-t) + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} t \right)$



(Different way to obtain a direction vector: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ are perpendicular to the planes

so $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix} = -3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$)

2. The curves $t\,\hat{\imath} + t^2\,\hat{\jmath} + t^3\,\hat{k}$ and $\sin(t)\,\hat{\imath} + \sin(2t)\,\hat{\jmath} + t\,\hat{k}$ intersect at the origin. Find the angle of intersection.

Tangent vectors: $\begin{bmatrix} 1 \\ 2t \\ 3t^2 \end{bmatrix}, \begin{bmatrix} \cos t \\ 2\cos(2t) \\ 1 \end{bmatrix}$

Eval. @ origin: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Dot product: $u \cdot v = |u||v| \cos \theta$

Here: $\quad 1 = \sqrt{6}\cos\theta$

$\therefore \cos\theta = \frac{1}{\sqrt{6}} \qquad \therefore \theta = \arccos\left(\frac{1}{\sqrt{6}}\right) = 1.15$ radians
$(65.9°)$

3. Find the limit of $xy^3/(x^4+2y^4)$ as $(x,y) \to (0,0)$ or show that the limit fails to exist.

$x = 0 \Rightarrow 0$

$x = y \Rightarrow \frac{y^4}{y^4+2y^4} = \frac{1}{3}$

} not equal, so the limit does not exist.

4. Suppose f is a differentiable function of x and y and $g(u,v) = f(e^u + \sin v, e^u + \cos v)$. Use the table of values to find the directional derivative of g at the origin along the main diagonal.

(x,y)	f	g	f_x	f_y
(0,0)	2	3	4	5
(1,2)	6	7	8	9

Chain Rule:

$$\nabla g = [g_u \ \ g_v] = [f_x \ \ f_y] \underbrace{\begin{bmatrix} x_u & x_v \\ y_u & y_v \end{bmatrix}}_{\begin{bmatrix} e^u & \cos v \\ e^u & -\sin v \end{bmatrix}}$$

$$\overset{\text{Eval}}{=} [8 \ \ 9] \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = [17 \ \ 8]$$

Direction: $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Directional derivative: $[17 \ \ 8]\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \boxed{\frac{25}{\sqrt{2}} = 17.7}$

5. Integrate $x/(1+xy)$ over the unit square $[0,1]\times[0,1]$.

$$\int_0^1 \left[\int_0^1 \frac{x}{1+xy}\,dy\right] dx = \int_0^1 \ln(1+xy)\Big|_0^1 dx$$

$$= \int_0^1 \ln(1+x)\,dx = x\ln(1+x)\Big|_0^1 - \int_0^1 x\,d(\ln(1+x))$$

$$= \ln 2 - \int_0^1 \frac{x}{1+x}\,dx = \ln 2 - \int_0^1 \left(1 - \frac{1}{1+x}\right) dx$$

$$= \ln 2 - x + \ln(1+x)\Big|_0^1 = \boxed{2\ln 2 - 1}$$