Midterm 1

1. An integer n is prime means $n > 1$ and the only positive divisors of n are 1 and itself.

   (a) Write this definition in formal language using appropriate quantifiers.

   (b) Negate the formal expression and simplify (show work)

   (c) Write out the negation in words.

1a $\quad n \in \mathbb{Z}$ is prime $\Leftrightarrow$

$n > 1 \wedge \forall a \in \mathbb{Z} \; (a > 0 \wedge a \mid n) \Rightarrow (a = 1 \vee a = n)$

1b $\quad n \in \mathbb{Z}$ is not prime $\Leftrightarrow$

$n \leq 1 \vee \exists a \in \mathbb{Z} \; a > 0 \wedge a \mid n \wedge a \neq 1 \wedge a \neq n$

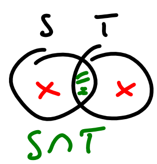
1c $\quad n \in \mathbb{Z}$ is not prime $\Leftrightarrow$

$n \leq 1$ or there is a positive divisor of n that is not 1 and not n :)

2. For each statement below determine whether it is true. If so, prove it (show work). If not, exhibit a concrete counterexample and explain why it is indeed a counterexample.

   (a) If S and T are sets, $S \cup T = S \cap T \Leftrightarrow S \subseteq T$

   (b) If S and T are sets, $S \cup T \subseteq S \cap T \Leftrightarrow S = T$

2a $\quad$ False. $\quad$ Venn diagram: 

so $S \cap T = S \cup T \Leftrightarrow S = T$

Counterexample: All we need is $S \subsetneq T$, so let $S = \emptyset$, $T = \{\emptyset\}$.

then $S \subseteq T$, but $S \cap T = \emptyset \neq T = S \cup T$

2b $\quad$ True. [$S \cap T \subseteq S \cup T$, so $S \cup T \subseteq S \cap T \Leftrightarrow S \cup T = S \cap T$] $(\Leftrightarrow S = T)$

Pf $\quad \Rightarrow$ If $x \in S$, $x \in S \cup T = S \cap T$, so $x \in T$. Sim. $x \in T \Rightarrow x \in S$:)

Alt: $S \subseteq S \cup T = S \cap T \subseteq T$. Sim. $T \subseteq S$, so $S = T$:)

$\Leftarrow \quad S \cup T = S \cup S = S = S \cap S = S \cap T$:)

3. Consider the Diophantine equation $54x - 28y = 8$

(a) Use extended Euclid's algorithm to find the greatest common divisor of 54 and -28 and to find a certificate for it (show work)

(b) Find the general integer solution to the equation.

(c) Find three distinct particular integer solutions to the equation.

3a Euclid:

$54 = 28 + 26 \qquad 26 = 54 - 28$

$28 = 26 + 2 \qquad 2 = 28 - 26 = 28 - (54 - 28) = \underline{(-1)}54 + \underline{(-2)}(-28)$

$26 = 13 \cdot 2$

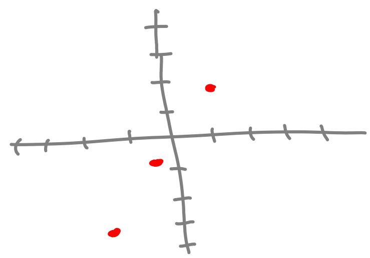
gcd

3b $x = (-1)\, 8/2 + n \frac{(-28)}{2} = -4 - 14n, \quad y = -2 \cdot \frac{8}{2} - n \frac{54}{2} = -8 - 27n$

3c $n = 0 \Rightarrow x = -4,\ y = -8$

$n = -1 \Rightarrow x = 10,\ y = 19$

$n = 1 \Rightarrow x = -18,\ y = -35$



Check in wxmaxima

```
(%i1) load("gcdex")$

(%i2) a:54;
```

(%o2) 54

```
(%i3) b:-28;
```

(%o3) -28

```
(%i4) c:8;
```

(%o4) 8

```
(%i5) [x1,y1,d]:-igcdex(a,b);
```

(%o5) $[-1,-2,2]$

```
(%i6) a*x1+b*y1;
```

(%o6) 2

```
(%i7) /* particular solution */
      x:x1*c/d;
```

(%o7) -4

```
(%i8) y:y1*c/d;
```

(%o8) -8

```
(%i9) a*x+b*y;
```

(%o9) 8

```
(%i10) [x+n*b/d,y-n*a/d];
```

(%o10) $[-14\,n-4,-27\,n-8]$

```
(%i11) create_list([x+n*b/d,y-n*a/d,a*(x+n*b/d)+b*(y-n*a/d)],n,-1,1);
```

(%o11) $[[10,19,8],[-4,-8,8],[-18,-35,8]]$

plot2d([discrete,create_list(x+n·b/d,n,−1,1),create_list(y−n·a/d,n,−1,1)],[style,points])$