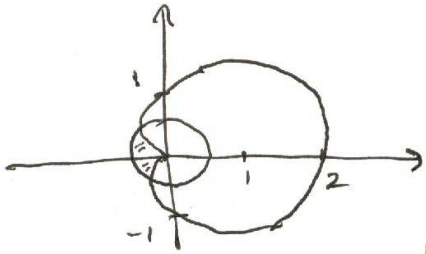
(1)



Find points of intersection:

$$\frac{1}{2} = 1 + \cos\theta$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \ldots, \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$$

Area of sector: $\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$ $\quad \therefore A = \frac{1}{2} r^2 \theta$

$$A = \frac{1}{2} \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left[\left(\frac{1}{2}\right)^2 - (1 + \cos\theta)^2 \right] d\theta = \boxed{0.2}$$

(2)

$$\frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}} \approx \frac{x^{1/4}}{x^{4/3}} = \frac{1}{x^{13/12}}$$

Since $\frac{13}{12} > 1$, the integral converges

Comparison: For all sufficiently large x

Say $x > a$ $\qquad \sqrt{x} + 1 < 4\sqrt{x}$

$$\therefore \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}} < \frac{\sqrt{4\sqrt{x}}}{\sqrt[3]{x^4}} = \frac{2}{x^{13/12}}$$

$$\therefore 0 \le \int_1^\infty \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}} dx = \int_1^a \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}} dx + \int_a^\infty \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}} dx$$

$$\le \underbrace{\int_1^a \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}} dx}_{< \infty} + \underbrace{\int_a^\infty \frac{dx}{x^{13/12}}}_{\text{conv.}} < \infty$$

(3) a)

x^2	$\cos(3x)$
$2x$ ⊕	$\frac{1}{3}\sin(3x)$
2 ⊖	$-\frac{1}{9}\cos(3x)$
0 ⊕	$-\frac{1}{27}\sin(3x)$

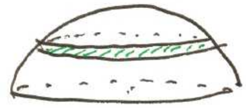
$$\int x^2\cos(3x)\,dx = \boxed{\frac{x^2}{3}\sin(3x) + \frac{2x}{9}\cos(3x) - \frac{2}{27}\sin(3x) + C}$$

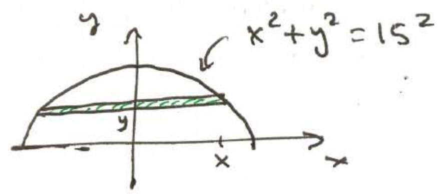
b)

$$\int \frac{x^2}{x^2+1}\,dx = \int\left(1 - \frac{1}{x^2+1}\right)dx$$

$$= \boxed{x - \arctan(x) + C}$$

(4)





$$\Delta V = \pi x^2 \Delta y = \pi(15^2 - y^2)\Delta y$$

$$\delta = 250 - \frac{100}{15}y$$

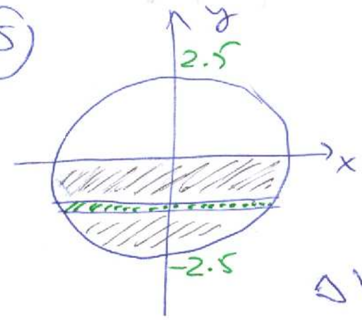
$$\Delta m = \delta\,\Delta V = \left(250 - \frac{100}{15}y\right)\pi(15^2 - y^2)\Delta y$$

$$M = \int \Delta m = \pi\int_0^{15}\left(250 - \frac{100}{15}y\right)(15^2 - y^2)\,dy = 1.5\times10^6$$

$$M_y = \int y\,dm = \pi\int_0^{15} y\left(250 - \frac{100}{15}y\right)(15^2 - y^2)\,dy = 7.82\times10^6$$

Center of mass (y coord) $= \dfrac{M_y}{M} = \boxed{5.2\text{ m}}$

(5)



As in the preceding problem

$$\Delta m = \delta \, \Delta V = 10^3 \pi (2.5^2 - y^2) \Delta y$$

$$\Delta W = \Delta F (2.5 - y) = \Delta m \, g \, (2.5 - y)$$

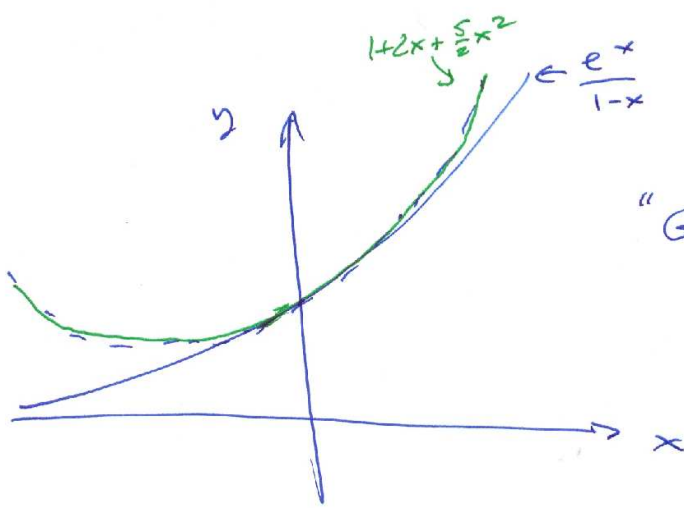
$$= 9.81 \cdot 10^3 \pi (2.5^2 - y^2)(2.5 - y) \Delta y$$

$$W = 9.81 \cdot 10^3 \pi \int_{-2.5}^{0} (2.5^2 - y^2)(2.5 - y) \, dy = \boxed{1.1 \times 10^6 \text{ joules}}$$

(6)

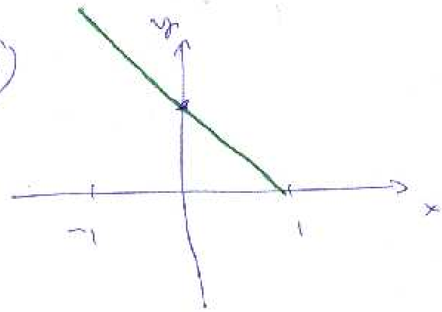
$$\frac{e^x}{1-x} = \left(1 + x + \frac{x^2}{2} + \dots\right)\left(1 + x + x^2 + \dots\right)$$

$$= 1 + 2x + \frac{5}{2} x^2 + \dots$$



"Good" on

$[-0.2, 0.2]$



(7)

$f(x) = 1 - x$ on $[-1, 1]$

$a_0 = \text{ave}(f) = 1$ (by inspection)

Also since $f - 1 = -x$ is odd

all $a_k = 0$ for $k \geq 1$.

$$b_k = \int_{-1}^{1} f(x) \sin(\pi k x)\, dx$$

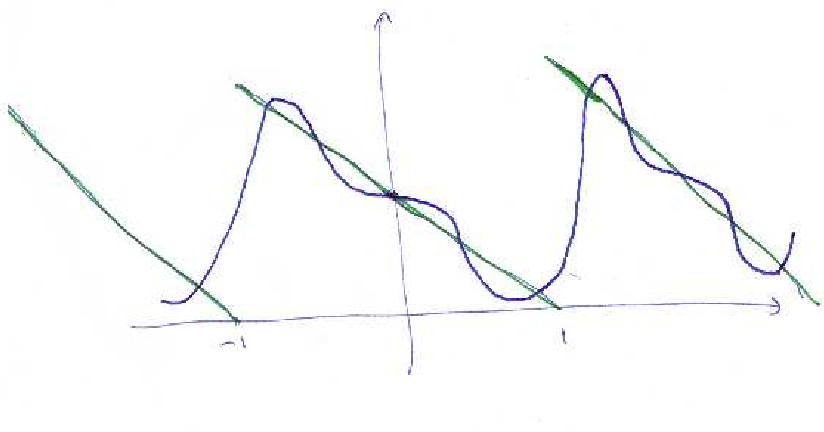
$1-x$	$\sin(\pi k x)$
-1 ⊕	$-\frac{1}{\pi k}\cos(\pi k x)$
0 ⊖	$-\frac{1}{\pi^2 k^2}\sin(\pi k x)$

$$= \left[-\frac{1-x}{\pi k}\cos(\pi k x) - \frac{1}{\pi^2 k^2}\sin(\pi k x) \right]_{-1}^{1}$$

$$= 0 - \left[-\frac{2}{\pi k}(-1)^k \right] = \frac{2}{\pi k}(-1)^k$$

$$b_1 = -\frac{2}{\pi} = -.6366, \quad b_2 = \frac{1}{\pi} = .3183$$

$$\therefore \boxed{f(x) \approx 1 - \frac{2}{\pi}\sin(\pi x) + \frac{1}{\pi}\sin(2\pi x)}$$



The approximation is the poorest @ $x = -1, 1$ Because f is discontinuous at these points

(8) $$\frac{dy}{dx} = y\, 2^x$$

$$\int \frac{dy}{y} = \int 2^x \, dx$$

$$\ln|y| = \frac{1}{\ln 2} 2^x + C$$

$$y = A e^{\frac{2^x}{\ln 2}}$$

Since $y(0) = 1$, $1 = A e^{\frac{1}{\ln 2}}$, so $A = e^{-\frac{1}{\ln 2}}$

$$\therefore y = e^{-\frac{1}{\ln 2}} e^{\frac{2^x}{\ln 2}} = e^{\frac{2^x - 1}{\ln 2}}$$

$$\therefore y(1) = e^{\frac{1}{\ln 2}} \boxed{= 4.232}$$