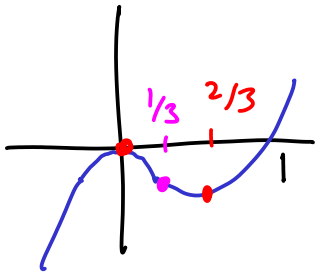
1. $f(x) = x^3 - x^2$



$f'(x) = 3x^2 - 2x$

$f''(x) = 6x - 2 = 2(3x-1)$

$f' = 0 \Rightarrow 3x^2 - 2x = 0, \quad x(3x-2) = 0$

crit. pts: $x = 0, \frac{2}{3}$

$f''(0) < 0 \quad \therefore \quad x = 0$ local max.

$f''\left(\frac{2}{3}\right) = 2\left(3 \cdot \frac{2}{3} - 1\right) > 0 \qquad x = 2/3$ local min

f incr. on $(-\infty, 0)$ and $\left(\frac{2}{3}, \infty\right)$

f decr on $(0, 2/3)$

$f'' = 0 \Rightarrow x = \frac{1}{3}$ infl. pt.

f is concave down on $\left(-\infty, \frac{1}{3}\right)$

—"— up on $\left(\frac{1}{3}, \infty\right)$

(2) $V = \pi r^2 h$ $(h = 10\text{ m})$

$dV = \pi 2rh\, dr$

$\frac{dV}{V} = \frac{\pi 2rh\, dr}{\pi r^2 h} = 2\frac{dr}{r}$ $\qquad \frac{dr}{r} = \frac{1}{2}\frac{dV}{V} = \frac{1}{2}\cdot 0.01$

$= 0.005$

$= 0.5\%$

Radius should be measured to 0.5% for volume to be within 1%.

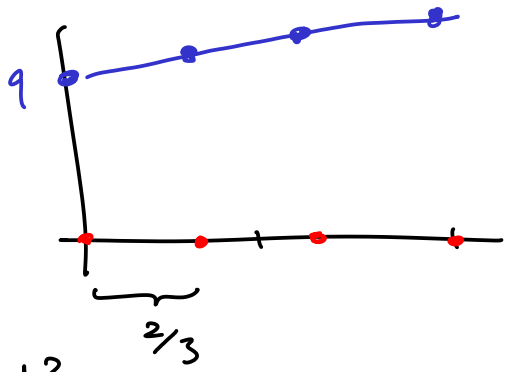
(3) Subdivide $[0, 2]$ into 3 subintervals:

$f(t) = 10 - 2^{-t}$ $\qquad t$: $0, \frac{2}{3}, \frac{4}{3}, 2$

$f(t)$: $9 \quad 9.37 \quad 9.6 \quad 9.75$

$L = \frac{2}{3}(9 + 9.37 + 9.6) = 18.65$

$U = R_3 = \frac{2}{3}(9.37 + 9.6 + 9.75) = 19.15$



$A = \int_0^2 (10 - 2^{-t})\, dt = 10t + \frac{2^{-t}}{\ln 2}\Big|_0^2 = 10\cdot 2 + \frac{1}{\ln 2}(2^{-2} - 1)$

$= 18.92$

$L < A < U$

$18.65 < 18.92 < 19.15$

At $t = 0$, 80 kg $\rightarrow$ At $t = 2$ $\quad 80 + 18.92$

$= 98.92$ kg

After 2 years the mycelium has mass 98.92 kg

(4) a) $\int_1^8 t^3 \sqrt[3]{t}\, dt = \int_1^8 t^3 t^{1/3}\, dt = \int_1^8 t^{10/3}\, dt$

$= \frac{t^{10/3+1}}{10/3+1}\Big|_1^8 = \frac{3}{13} t^{13/3}\Big|_1^8 = \frac{3}{13}(2^{13}-1)$

$= \frac{24573}{13} = 1890.23$

b) $\int_{-\frac{\pi}{2}}^{\pi} \sin(6t)\, dt = -\frac{1}{6}\cos(6t)\Big|_{-\frac{\pi}{2}}^{\pi} =$

1st Guess: $\cos(6t)$

diff: $-\sin(6t)\cdot 6$

$-\frac{1}{6}\left[1-(-1)\right] = -\frac{1}{3}$

Subst: Let $u = 6t$

Then $du = 6\, dt$

$dt = \frac{1}{6} du$

$\frac{1}{6}\int \sin u\, du$

$= -\frac{1}{6}\cos u$

(5) a) $\int \frac{1+t}{t}\, dt = \int \left(\frac{1}{t}+1\right) dt = \ln|t| + t + C$

b) $\int 5^{2t}\, dt = \frac{1}{2\ln 5} 5^{2t} + C$

1st guess 5^{2t}

diff: $\ln 5\ 5^{2t}\cdot 2$