Biocalc (Honors Cal I) Sp. 2006 Midterm 2

(1) $$2^{\cosh y} = y \ln x$$

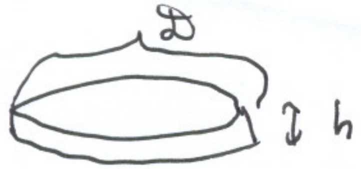
$\downarrow d$

$$\ln 2 \cdot 2^{\cosh y} \cdot \sinh y \cdot dy = dy \cdot \ln x + y \cdot \frac{1}{x} \cdot dx$$

$$\left(\ln 2 \cdot 2^{\cosh y} \sinh y - \ln x\right) dy = \frac{y}{x}\, dx$$

$$\boxed{\frac{dy}{dx} = \frac{y}{x\left(\ln 2 \cdot 2^{\cosh y} \sinh y - \ln x\right)}}$$

(3)



$h = 0.25\,\text{cm}$

$$V = \pi r^2 h = \pi \left(\frac{D}{2}\right)^2 h = \frac{\pi h}{4} D^2 = \frac{\pi}{16} D^2$$

$\downarrow \frac{d}{dt}$

$$\frac{dV}{dt} = \frac{\pi}{16} 2D \frac{dD}{dt} = \frac{\pi}{8} D \frac{dD}{dt}$$

Solve: $$\frac{dD}{dt} = \frac{8}{\pi D} \frac{dV}{dt} = \frac{8}{\pi 4} \cdot 1 = \frac{2}{\pi} \approx 0.6$$

$\therefore$ the diameter grows at the rate of about 0.6 cm/year

(2) $f = x e^{ax}$

$f' = e^{ax} + x e^{ax} \cdot a$

$= e^{ax}(1 + xa)$, $\quad f'(2) = e^{2a}(1+2a)$

$f' = 0 \Rightarrow \boxed{a = -\frac{1}{2}.}$

$f'' = e^{ax} a (1 + xa) + e^{ax} \cdot a$

$= a e^{ax}(2 + xa)$

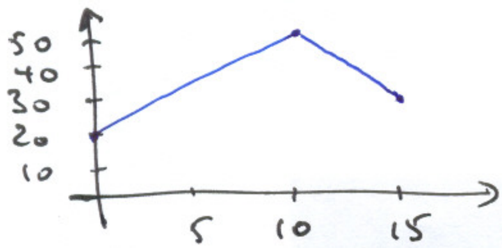
$a = -\frac{1}{2} \Rightarrow f'' = -\frac{1}{2} e^{-\frac{1}{2}x}(2 - \frac{x}{2})$

$f''(2) = -\frac{1}{2}\frac{1}{e} < 0$

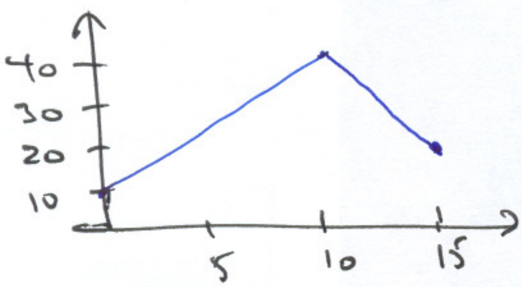
$\therefore$ f is concave down

$\therefore$ $\boxed{\text{f has a local max @ } x = 2.}$

④ Rate of delivery:



Include clearance:

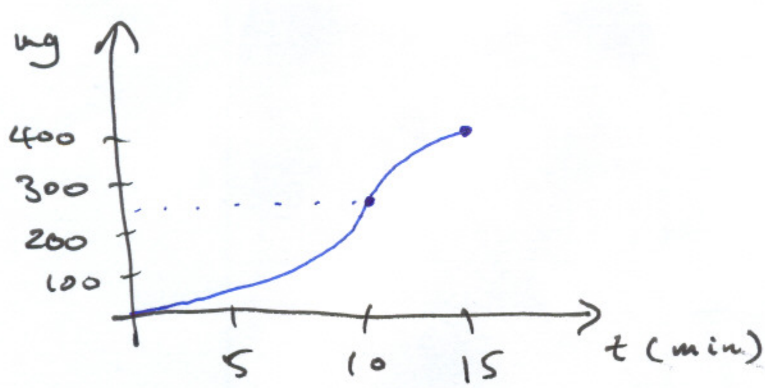


$$\text{Area} = 10 \cdot \frac{10+40}{2} + 5\,\frac{40+20}{2}$$

$$= 250 + 150 = 400$$

$\therefore$ At the end the patient has 400 mg of the drug

Sketch:



⑤ a) $\displaystyle\int_1^4 \frac{1+t}{\sqrt{t}}\,dt = \int_1^4 \left(t^{-\frac{1}{2}} + t^{\frac{1}{2}}\right) dt$

$$= 2t^{\frac{1}{2}} + \frac{2}{3}t^{\frac{3}{2}} \Big|_1^4 = 2\cdot 4^{\frac{1}{2}} + \frac{2}{3}4^{\frac{3}{2}} - 2 - \frac{2}{3}$$

$$= 2\cdot 2 + \frac{2}{3}\cdot 2^3 - 2 - \frac{2}{3} = 2 + \frac{2}{3}(8-1) = 2 + \frac{14}{3}$$

$$= \frac{6+14}{3} = \boxed{\frac{20}{3}}$$

b) $\displaystyle\int_0^1 \sin(\pi t)\,dt = -\frac{\cos(\pi t)}{\pi}\Big|_0^1$

$$= -\frac{1}{\pi}\left[-1-1\right] = \boxed{\frac{2}{\pi}}$$