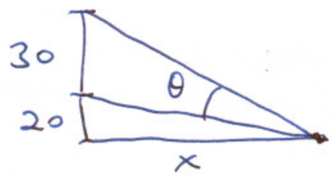
Midterm 2 Cal I (honors) MAT 1214.004 Fall 2005

(1)



$$\theta = \arctan\left(\frac{50}{x}\right) - \arctan\left(\frac{20}{x}\right)$$

$$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{50}{x}\right)^2}\left(-\frac{50}{x^2}\right) - \frac{1}{1+\left(\frac{20}{x}\right)^2}\left(-\frac{20}{x^2}\right)$$

$$= -\frac{50}{x^2+50^2} + \frac{20}{x^2+20^2} = \frac{-50(x^2+20^2)+20(x^2+50^2)}{(x^2+50^2)(x^2+20^2)}$$

$$\frac{d\theta}{dx} = 0 \Rightarrow -30x^2 + 20\cdot 50^2 - 50\cdot 20^2 = 0$$

$$30x^2 = 50\cdot 20\,(50-20)$$

$$30x^2 = 1000\cdot 30$$

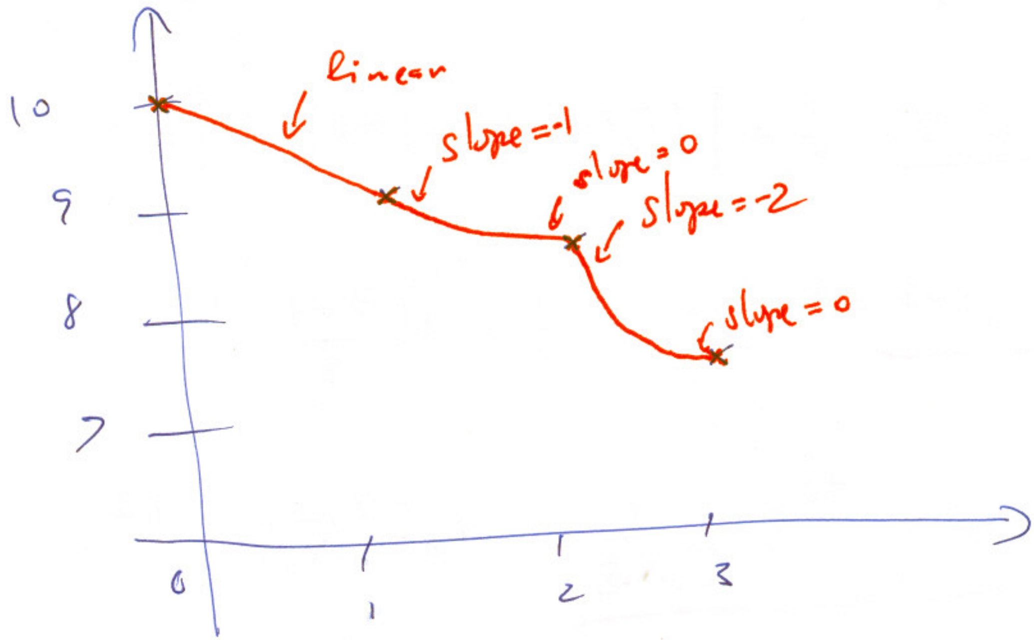
$$x^2 = 1000$$

$$x = \sqrt{1000} = 10\sqrt{10} \approx 31.6$$

∴ the optimal viewing distance is about <u>31.6 meters</u>

(2) First compute area and construct a table of values

t	volume
0	10
1	9
2	8.5
3	7.5



(3)

a) $$\int_1^4 t^2\sqrt{t}\,dt = \int_1^4 t^{5/2}\,dt = \frac{2}{7}\,t^{\frac{7}{2}}\Big|_1^4$$

$$= \frac{2}{7}\left[4^{7/2} - 1^{7/2}\right] = \frac{2}{7}\left[2^7 - 1\right] = \frac{2}{7}\cdot 127$$

$$= \frac{254}{7} = \underline{36.2857}$$

b) $$\int_{-\frac{\pi}{2}}^{\pi} \cos(5t)\,dt = \frac{1}{5}\sin(5t)\Big|_{-\frac{\pi}{2}}^{\pi}$$

$$= \frac{1}{5}\left[\sin(5\pi) - \sin\left(-\frac{5\pi}{2}\right)\right] = \frac{1}{5} = \underline{0.2}$$

c) $$\int \frac{1+t}{t}\,dt = \int \left[\frac{1}{t} + 1\right] dt = \underline{\ln|t| + t + C}$$

d) $$\int 7^{3t}\,dt = \underline{\frac{1}{3\ln 7}\,7^{3t} + C}$$

(4) Let $p(t)$ denote the # of casualties by time t.

Then $\frac{dp}{dt} = kt$, where $k > 0$ is a constant.

$\therefore$ $p = \frac{kt^2}{2} + C$.

Initially there are no casualties so $C = 0$.

so $p = \frac{kt^2}{2}$

$p(1) = 100{,}000 - 98{,}000 = 2{,}000$

so $\frac{k}{2} = 2{,}000$ so $p = 2{,}000t^2$

$p(5) = 50{,}000$

$\therefore$ after 5 years the army would have $100{,}000 - 50{,}000 = 50{,}000$ soldiers.

Half gone! ☹