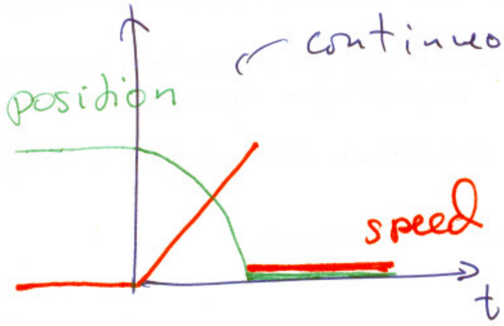
1)



continuous (since you don't materialize instantaneously far away)

not continuous since speed drops practically instantaneously to zero when you hit the floor

2) a)

$$\lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

$$\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} (-1) = -1$$

Since the one sided limits are not equal the limit does not exist.

b)

$$\lim_{x \to \infty} \frac{1-x^2}{1+2x+3x^2} = \lim_{x \to \infty} \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + \frac{2}{x} + 3}$$

$$= \frac{\lim_{x \to \infty} \frac{1}{x^2} - \lim_{x \to \infty} 1}{\lim_{x \to \infty} \frac{1}{x^2} + \lim_{x \to \infty} \frac{2}{x} + 3} = \frac{0-1}{0+0+3} = \boxed{-\frac{1}{3}}$$

③ a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} =$

$$= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{\cancel{(x+h)} - \cancel{x}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\lim\limits_{h\to 0}\sqrt{x+h} + \lim\limits_{h\to 0}\sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

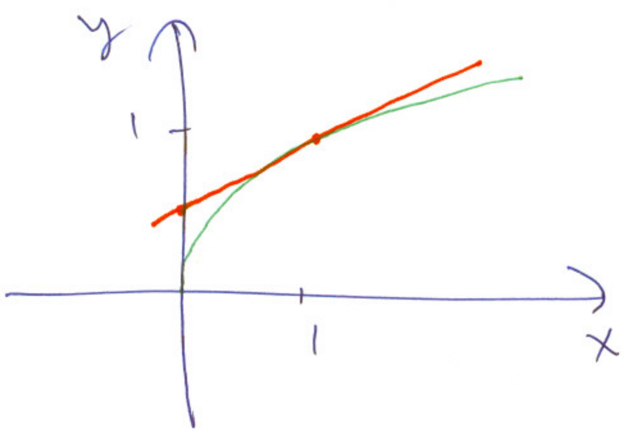
By the power rule $\left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$

The answers agree.

b) Linear approximation: $y = f(a) + f'(a)(x-a)$

$f(1) = \sqrt{1} = 1 \qquad f'(1) = \dfrac{1}{2\sqrt{1}} = \dfrac{1}{2}$

$\therefore \quad y = 1 + \frac{1}{2}(x-1) = \boxed{\frac{1}{2}x + \frac{1}{2}}$



(4) If $y = \arctan(x^2)$

$$y' = \frac{1}{1+(x^2)^2}(x^2)' = \frac{2x}{1+x^4}$$

$$y'' = 2\,\frac{x'(1+x^4) - x(1+x^4)'}{(1+x^4)^2}$$

$$= 2\,\frac{1+x^4 - x\cdot 4x^3}{(1+x^4)^2} = 2\,\frac{1-3x^4}{(1+x^4)^2}$$

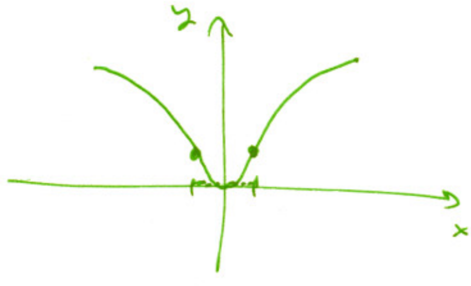
$x^4 \geq 0$ so $1+x^4 > 0$ so $(1+x^4)^2 > 0$

$\therefore$ $y'' > 0$ when $1 - 3x^4 > 0$

i.e. $3x^4 < 1$, $x^4 < \frac{1}{3}$,

$$|x| < \frac{1}{3^{1/4}} = 0.76$$

$$\boxed{-\frac{1}{3^{1/4}} < x < \frac{1}{3^{1/4}}}$$



(5)

$$2^x \ln(y) = \cos(xy)$$

$$d(2^x)\ln(y) + 2^x\, d(\ln y) = -\sin(xy)\, d(xy)$$

$$2^x \ln 2\, dx \cdot \ln y + 2^x \frac{1}{y}\, dy = -\sin(xy)\left[dx \cdot y + x\,dy\right]$$

$$dy\left[\frac{2^x}{y} + x\sin(xy)\right] = -dx\left[y\sin(xy) + 2^x \ln 2 \ln y\right]$$

$$\therefore \boxed{\frac{dy}{dx} = -\frac{y\sin(xy) + 2^x \ln 2 \ln y}{\frac{2^x}{y} + x\sin(xy)}}$$