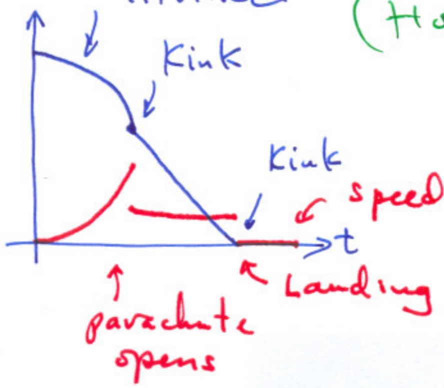
Cal I (Honors) MAT 1214.004 Fall 2005 Final

(1)



altitude - continuous (no teleportation ⌣)

speed not continuous when the parachute pops open and when Betty hits the ground.

(2) a) $\dfrac{1-x}{1-x^3} = \dfrac{\frac{1}{x^3}-\frac{1}{x^2}}{\frac{1}{x^3}-1} \to \dfrac{0-0}{0-1} = \boxed{0}$ as $x \to \infty$

b) $\dfrac{1-x}{1-x^3} = \dfrac{1-x}{(1-x)(1+x+x^2)} = \dfrac{1}{1+x+x^2} \to \dfrac{1}{1+1+1} = \boxed{\dfrac{1}{3}}$ as $x \to 1$

(3) a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim\limits_{h\to 0} \dfrac{x^2-(x+h)^2}{(x+h)^2x^2h}$

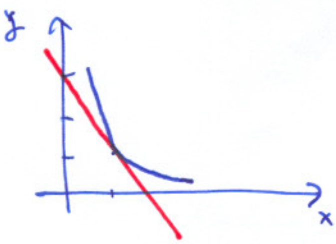
$= \lim\limits_{h\to 0} \dfrac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{(x+h)^2x^2h} = \lim\limits_{h\to 0} \dfrac{-2x\cancel{h}-h^{\cancel{2}}}{(x+h)^2x^2\cancel{h}} = \lim\limits_{h\to 0} \dfrac{-2x-h}{(x+h)^2x^2}$

$= \dfrac{-2x-0}{(x+0)^2x^2} = \dfrac{-2x}{x^4} = \boxed{-\dfrac{2}{x^3}}$

Power rule: $\left(\frac{1}{x^2}\right)' = (x^{-2})' = -2x^{-2-1} = -2x^{-3} = \dfrac{-2}{x^3}$

b) Tangent line:

$y = f(1) + f'(1)(x-1) = 1 - 2(x-1) = \boxed{-2x+3}$



(4) $y = \frac{1}{1+x^2}$, $y' = -\frac{1}{(1+x^2)^2} \cdot 2x = -2\frac{x}{(1+x^2)^2}$

$$y'' = -2\frac{x'(1+x^2)^2 - x[(1+x^2)^2]'}{(1+x^2)^4} = -2\frac{(1+x^2)^{\cancel{2}} - x\,2\cancel{(1+x^2)}\cdot 2x}{(1+x^2)^{\cancel{4}\,3}}$$

$$= -2\,\frac{1+x^2-4x^2}{(1+x^2)^3} = -2\frac{1-3x^2}{(1+x^2)^3}$$

The bottom is >0, so $y''<0$ when $1-3x^2>0$

i.e. when $3x^2<1$, $x^2<\frac{1}{3}$, $\boxed{-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}}$

(5) $\pi^y \arctan(2x) = \cosh(x^2y)$, take d

$$d(\pi^y)\arctan(2x) + \pi^y\, d(\arctan(2x)) = \sinh(x^2y)\, d(x^2y)$$

$$\ln(\pi)\,\pi^y \arctan(2x)\,dy + \pi^y\frac{1}{1+(2x)^2}\cdot 2\,dx = \sinh(x^2y)(2xy\,dx + x^2dy)$$

Collect terms:

$$dy\left[\ln(\pi)\,\pi^y\arctan(2x) - \sinh(x^2y)\,x^2\right]$$
$$= dx\left[2xy\sinh(x^2y) - \frac{2\pi^y}{1+4x^2}\right]$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{2xy\sinh(x^2y) - \frac{2\pi^y}{1+4x^2}}{\ln(\pi)\,\pi^y\arctan(2x) - x^2\sinh(x^2y)}}$$

(6) Revenue $R = pq = p(1500 - 50p)$

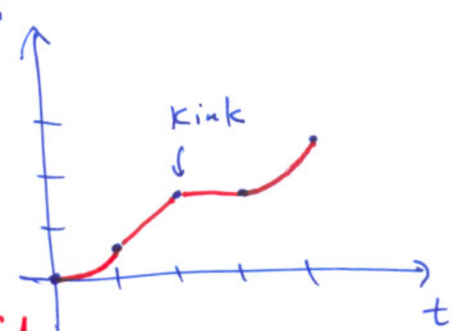
$= 1500p - 50p^2$ $\xrightarrow{\text{graph}}$

$\frac{dR}{dp} = 1500 - 50 \cdot 2p = 1500 - 100p$

$\frac{dR}{dp} = 0 \Rightarrow p = 15$ $\therefore$ Software should be priced @ $\boxed{\$15}$

(7) Table:

time	volume
0	0
1	0.5
2	1.5
3	1.5
4	2.5



After 4 hrs we have $\boxed{250 \text{ gallons}}$.

(8) a) $\int_1^4 \frac{1}{t^2\sqrt{t}}\,dt = \int_1^4 t^{-2.5}\,dt = \left.\frac{t^{-1.5}}{-1.5}\right|_1^4 = -\frac{2}{3}\left.t^{-\frac{3}{2}}\right|_1^4$

$= -\frac{2}{3}\left[4^{-\frac{3}{2}} - 1^{-\frac{3}{2}}\right] = -\frac{2}{3}\left[2^{-3} - 1\right] = -\frac{2}{3}\left[\frac{1}{8} - 1\right] = \frac{2}{3}\,\frac{7}{8} = \boxed{\frac{7}{12}}$

b) $\int_{-\frac{\pi}{2}}^{\pi} \sin(7t)\,dt = \left.-\frac{\cos(7t)}{7}\right|_{-\frac{\pi}{2}}^{\pi} = -\frac{1}{7}\left[\cos(7\pi) - \cos\left(-\frac{7\pi}{2}\right)\right]$

$= -\frac{1}{7}\left[-1 - 0\right] = \boxed{\frac{1}{7}}$

c) $\int \frac{1+t}{t}\,dt = \int \left(\frac{1}{t} + 1\right)dt = \boxed{\ln|t| + t + C}$

d) $\int \pi^{et}\,dt = \boxed{\frac{1}{e\ln(\pi)}\pi^{et} + C}$

(9) Let s = diameter of tumor at time t

then $\frac{ds}{dt} = k\sqrt{t} = k t^{1/2}$, where k is a constant ($k > 0$)

$s(0) = 2$, $s(1) = 3$, $s(12) = ?$

$$s = k \frac{t^{1/2+1}}{1/2+1} = \frac{2k}{3} t^{\frac{3}{2}} + \underset{\substack{\parallel \\ 2}}{C} = \frac{2k}{3} t^{\frac{3}{2}} + 2$$

since $s(0) = 2$

$s(1) = 3 \Rightarrow 3 = \frac{2k}{3} + 2$, so $\frac{2k}{3} = 1$ so

$s = t^{\frac{3}{2}} + 2$, so $s(12) = 12^{3/2} + 2 \approx 43.57$

$\therefore$ After a year the tumor is $\boxed{43.57 \text{ mm}}$ in diameter. Huge! :)

Have a great break!

(see you next year)

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