MAT 1214.2 Fall 2001 Midterm 2

① a) $y=[\cos\sqrt{1-x^2}]^3$, $dy=3[\cos\sqrt{1-x^2}]^2(-\sin\sqrt{1-x^2})\frac{1}{2\sqrt{1-x^2}}(-2x)dx$

b) Product Rule: $d[(x^2y+1)^7](x-y^3)^8+(x^2y+1)^7 d[(x-y^3)^8]=0$

$7(xy^2+1)^7(dx\cdot y^2+x2ydy)(x-y^3)^8+(xy^2+1)^7 8(x-y^3)^7(dx-3y^2dy)=0$

Now Solve for dy. First collect terms:

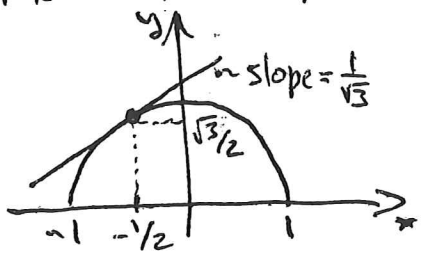
$$[7y^2(xy^2+1)^6(x-y^3)^8+8(xy^2+1)^7(x-y^3)^7]dx+$$
$$+[14xy(xy^2+1)^6(x-y^3)^8-24y^2(xy^2+1)^7(x-y^3)^7]dy=0$$

$$dy=-\frac{7y^2(xy^2+1)^6(x-y^3)^8+8(xy^2+1)^7(x-y^3)^7}{14xy(xy^2+1)^6(x-y^3)^8-24y^2(xy^2+1)^7(x-y^3)^7}dx$$

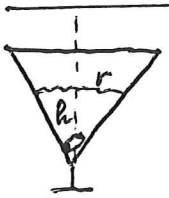
② $f(x)=\sqrt{1-x^2}$, $f(-\frac{1}{2})=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$, $f'(x)=\frac{1}{2\sqrt{1-x^2}}(-2x)=-\frac{x}{\sqrt{1-x^2}}$, $f'(-\frac{1}{2})=-\frac{-1/2}{\sqrt{3}/2}=\frac{1}{\sqrt{3}}$

$f(x)\approx f(a)+f'(a)(x-a)=\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}(x+\frac{1}{2})=$

$=\frac{1}{\sqrt{3}}x+(\frac{\sqrt{3}}{2}+\frac{1}{2\sqrt{3}})\approx 0.577x+1.15$



③



a) Total volume: $\frac{1}{3}\pi 5^2 7-1\approx 182\ cm^3$

b) Time = volume/rate = $\frac{182}{2}=91$ sec

c) $V=\frac{1}{3}\pi r^2 h-1$, $r=\frac{5}{7}h$, $V=\frac{1}{3}\pi\frac{5^2}{7^2}h^3-1$

Approx: $V=0.534h^3-1$

d) Set $V=182/2=91$ and solve for h:

$$h=\sqrt[3]{\frac{V+1}{0.534}}=\left(\frac{92}{0.534}\right)^{1/3}=5.56\ cm$$

e) $\underbrace{\frac{dV}{dt}}_{2}=0.534\cdot 3h^2\frac{dh}{dt}$, $h=5.56$

Solve for $\frac{dh}{dt}$: $\frac{dh}{dt}=\frac{2}{0.534\cdot 3\cdot 5.56^2}=0.04\ \frac{cm}{sec}$