1. A population of four million bacteria is introduced into petri dish and grows exponentially, doubling in size every seven hours. Write down the size of the culture as a function of time. When will the colony reach fifty million?

$b(t) = 4 \cdot 2^{t/7}$

If $50 = 4 \cdot 2^{t/7}$, then $12.5 = 2^{t/7}$,

so $\ln 12.5 = \ln 2^{t/7} = \frac{t}{7} \ln 2$, so

$t = 7 \cdot \frac{\ln 12.5}{\ln 2} = 25.5$, so the colony will reach 50 million after 25.5 hrs

```
> b:=4*2^(t/7);
```

$$b := 4\,2^{\left(\frac{t}{7}\right)}$$

```
> solve(b=50,t); evalf(%);
```

$$\frac{7\ln\left(\frac{25}{2}\right)}{\ln(2)}$$

$$25.50699332$$

2. The level of a hormone varies according to $s(t) = 3 + 2\sin(0.5t)$ where time t is measured in months. Find and illustrate on a graph

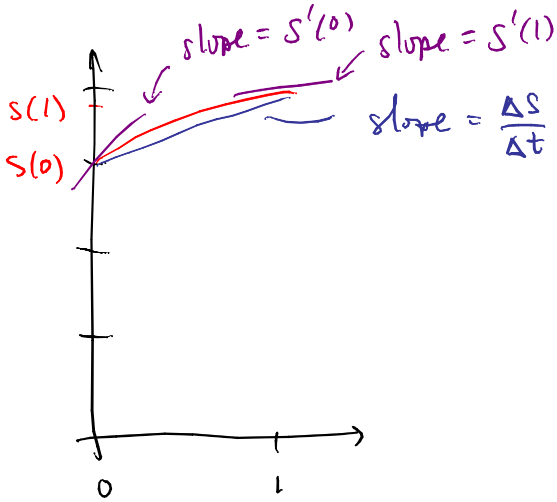
(a) Initial size and the size after a month.
(b) The instantaneous rates of change at those two times.
(c) The average rate of change during that period of time.

a) $s(0) = 3$, $s(1) = 3 + 2\sin 0.5 = 3.96$

b) $s'(t) = 2\cos(0.5t)0.5 = \cos(0.5t)$

$s'(0) = 1$, $s'(1) = \cos(0.5) = 0.88$

c) $\frac{\Delta s}{\Delta t} = \frac{3.96 - 3}{1 - 0} = 0.96$




```
> s:=3+2*sin(0.5*t);
```

$$s := 3 + 2\sin(0.5\,t)$$

```
> subs(t=0,s); s0:=evalf(%);
  subs(t=1,s); s1:=evalf(%);
```

$$3 + 2\sin(0.)$$
$$s0 := 3.$$
$$3 + 2\sin(0.5)$$
$$s1 := 3.958851077$$

```
> ave:=(s1-s0)/(1-0);
```

$$ave := 0.958851077$$

```
> ds:=diff(s,t);
```

$$ds := 1.0\cos(0.5\,t)$$

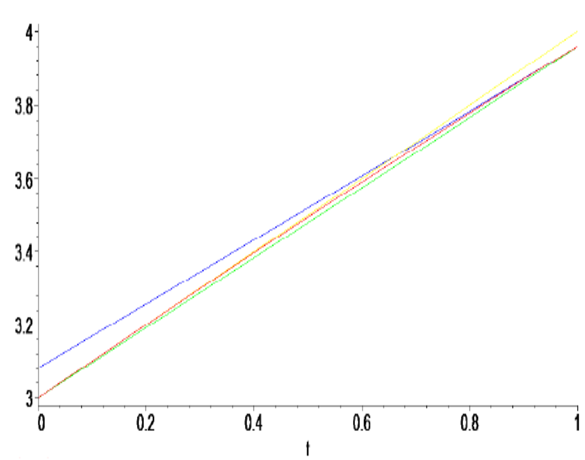
```
> subs(t=0,ds); ds0:=evalf(%);
  subs(t=1,ds); ds1:=evalf(%);
```

$$1.0\cos(0.)$$
$$ds0 := 1.0$$
$$1.0\cos(0.5)$$
$$ds1 := 0.8775825619$$

```
> secant:=s0+ave*(t-0);
  tangent0:=s0+ds0*(t-0);
  tangent1:=s1+ds1*(t-1);
```

$$secant := 3. + 0.958851077\,t$$
$$tangent0 := 3. + 1.0\,t$$
$$tangent1 := 3.081268515 + 0.8775825619\,t$$

```
> plot({s,secant,tangent0,tangent1},t=0..1);
```



3. Find the derivatives of

(a) 2^{2^t} (b) 2^{t^2}

$(a^t)' = \ln a \; a^t$

```
> 2^(2^t); diff(%,t);
  2^(t^2); diff(%,t);
```

$$2^{(2^t)}$$
$$2^{(2^t)}\,2^t \ln(2)^2$$
$$2^{(t^2)}$$
$$2\,2^{(t^2)} \ln(2)\, t$$

a) $\left(2^{2^t}\right)' = \ln 2 \cdot 2^{2^t} \cdot \ln 2 \cdot 2^t = (\ln 2)^2\, 2^{2^t + t}$

b) $\left(2^{t^2}\right)' = \ln 2 \cdot 2^{t^2} \cdot 2t$

4. Find the second derivative of $f(t) = te^{-t}$ and use it to describe the curvature of the graph of f for $t \geq 0$.

$f' = t'e^{-t} + t(e^{-t})' = e^{-t} + te^{-t}(-1) = e^{-t}(1-t)$

$f'' = (e^{-t})'(1-t) + e^{-t}(1-t)' = e^{-t}(-1)(1-t) + e^{-t}(-1)$

$= e^{-t}(t-2)$

$f'' < 0$ for $t < 2$ (f is concave down)

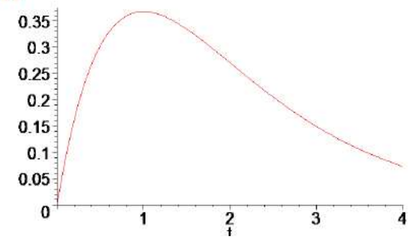
$f'' > 0$ for $t > 2$ (f is concave up)

$t = 2$ is a point of inflection.

```
> f:=t*exp(-t);
  diff(%,t); factor(%);
  diff(%,t); factor(%);
```

$$f := t\,e^{(-t)}$$
$$e^{(-t)} - t\,e^{(-t)}$$
$$-e^{(-t)}(t-1)$$
$$e^{(-t)}(t-1) - e^{(-t)}$$
$$e^{(-t)}(t-2)$$

```
> plot(f,t=0..4);
```



5. A population x_t has *per capita* production $0.5x_t$. Write down the discrete dynamical system for x_t. Find equilibria and use the slope criterion to determine their stability. Describe in words what happens in the long run.

$x_{t+1} = 0.5x_t^2$ updating function $f(x) = 0.5x^2$

Equilibria: $x = f(x) = 0.5x^2 \quad \therefore x = 0$ or $1 = 0.5x$ so $x = 2$

$x = 0, 2$

$f'(x) = 0.5 \cdot 2x = x$

$f'(0) = 0 < 1$ so $x = 0$ is stable

$f'(2) = 2 > 1$ so $x = 2$ is unstable

In the long run, if the initial population $x_0 < 2$, we have extinction. If $x_0 > 2$, we have population explosion!

```
> f:=0.5*x^2;
```

$$f := 0.5\,x^2$$

```
> equi:=[solve(f=x,x)];
```

$$equi := [0., 2.]$$

```
> df:=diff(f,x);
```

$$df := 1.0\,x$$

```
> map(v->subs(x=v,df),equi);
```

$$[0., 2.0]$$

```
> plot({f,x},x=0..2.5);
```