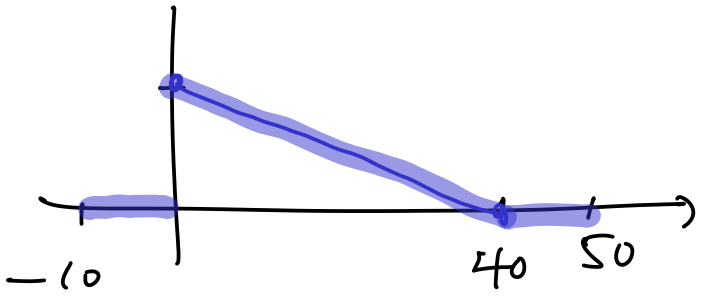
① $p(t) = \begin{cases} c(1 - 0.025t) & 0 \le t \le 40 \\ 0 & \text{otherwise} \end{cases}$

a) $p(0) = c \qquad p(40) = 0$



$1 = \text{Area} = \frac{1}{2} 40 \cdot c$

$\therefore c = \frac{1}{20} = 0.05$

b) $P(t \le 10) = \int_0^{10} 0.05(1 - 0.025t)\, dt$

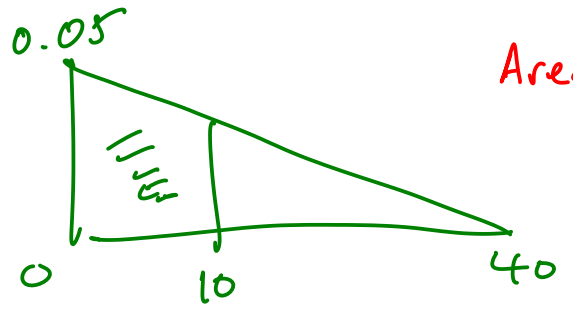
$= \underbrace{0.05\left(t - 0.025\,\frac{t^2}{2}\right)}_{cdf} \Big|_0^{10} = cdf(10)$

$= 0.05\left(10 - 0.0125 \cdot 10^2\right)$

$= 0.5\,(1 - 0.125) = 0.4375$

Checks: 1. $cdf(40) = 0.05\left(40 - 0.0125 \cdot 40^2\right) = 1$

2.



$$\text{Area} = 10\,\frac{0.05 + 0.05(1 - 0.025 \cdot 10)}{2}$$

$$= 0.25(1 + 1 - 0.25)$$

$$= 0.25 \cdot 1.75 = 0.4375$$

$\therefore$ 43.7% chance that a tattoo will heal within 1st ten days.

c) $$\mu = \int_{-\infty}^{\infty} t\,p(t)\,dt = \int_{0}^{40} t\,0.05(1 - 0.025t)\,dt$$

$$= 0.05 \int_{0}^{40} (t - 0.025t^2)\,dt$$

$$= 0.05 \left[\frac{t^2}{2} - 0.025\,\frac{t^3}{3} \right]_{0}^{40}$$

$$= 0.05 \left[\frac{40^2}{2} - 0.025\,\frac{40^3}{3} \right] = 13.333\ldots$$

$\therefore$ On average a tattoo will take 13.3 days to heal.

d) Set $cdf(t) = \frac{1}{2}$, Solve for t.

$$0.05\left(t - 0.0125 \cdot t^2\right) = 0.5$$

$$t - 0.0125\,t^2 = 10$$

$$0.0125t^2 - t + 10 = 0$$

$$t = \frac{1 \pm \sqrt{1 - 4 \cdot 0.0125 \cdot 10}}{2 \cdot 0.0125}$$

$$= \cancel{68.28}, \boxed{11.7}$$

∴ Half the tattoos will heal By 11.7 days

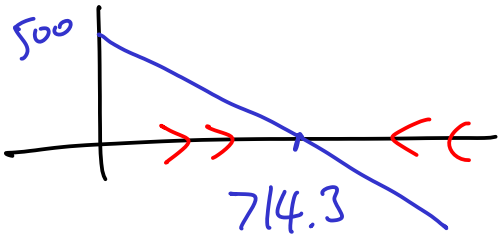
(2)

a) $\frac{dy}{dt} = 500 - 0.7y$

b) Equi $\Rightarrow \frac{dy}{dt} = 0 \Rightarrow 500 - 0.7y = 0$

$$y = \frac{500}{0.7} = \boxed{714.3}$$

$\therefore$ There is one equilibrium : 714.3 mg

c) 

$\therefore$ 714.3 mg is a stable equilibrium

d) In the long run the amount of drug in the body stabilizes at 714.3 mg

(3) $\frac{dB}{dt} = \frac{3B}{t}$ $\int \frac{dB}{B} = 3\int \frac{dt}{t}$

$\ln|B| = 3\ln|t| + C$

Since $B, t > 0$ $\ln B = 3\ln t + C$

$B = e^{3\ln t + C} = e^C \cdot e^{3\ln t} =$

$= e^C (e^{\ln t})^3 = \underbrace{e^C}_{k} t^3 = kt^3$

$\therefore$ General solution $\boxed{B = kt^3}$

$B(0) = 0 \neq 4$, so no particular solution. ☹