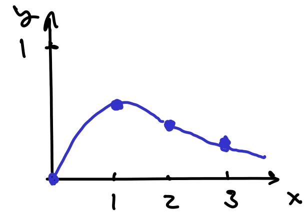
MAT 1193.003 Midterm 2 Fall 2014

① a)

x	0	1	2	3	$\to\infty$
y	0	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$	$\to 0$



$y = \dfrac{x}{1+x^2}$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

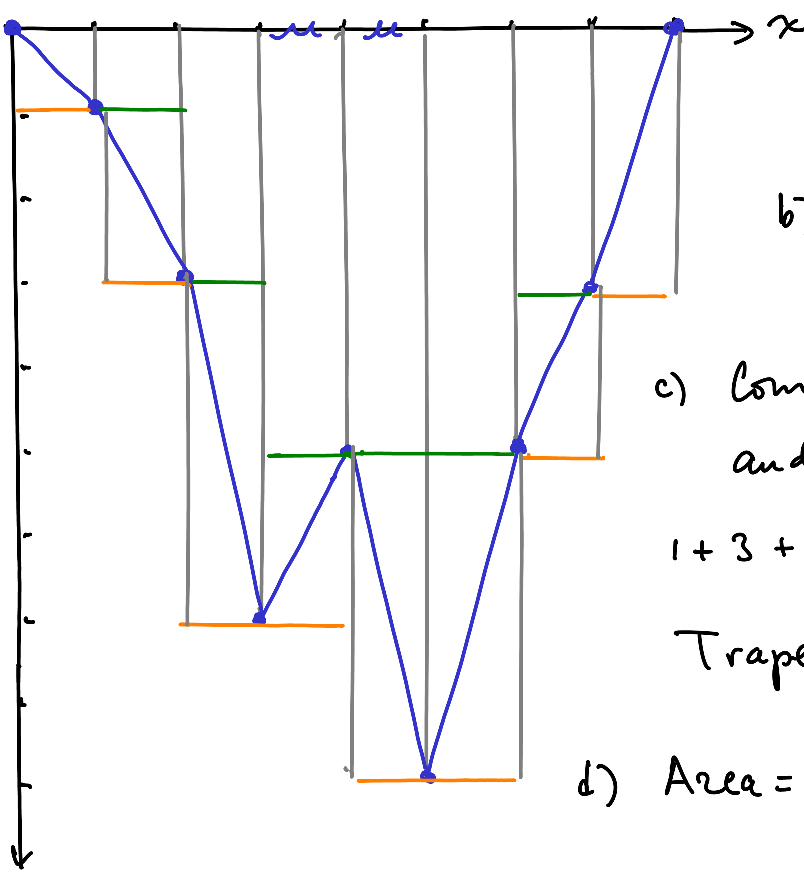
$$\frac{dy}{dx} = \frac{x'(1+x^2) - x(1+x^2)'}{(1+x^2)^2} = \frac{1+x^2 - x\cdot 2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$\therefore$ For $x \geq 0$ max $y = \frac{1}{2}$ at $\boxed{x = 1}$

b) The maximum fraction cancer free is half. It is achieved with dosage 1 mg

② a)



b) Lower estimate (green)

$1 + 3 + 5 + 5 + 5 + 3 = \boxed{22 \text{ ft}^2}$

c) Compute upper estimate (orange) and average (trapezoidal)

$1 + 3 + 7 + 7 + 9 + 9 + 5 + 3 = 44 \text{ ft}^2$

$$\text{Trapezoidal} = \frac{44 + 22}{2} = \boxed{33 \text{ ft}^2}$$

d) $\text{Area} = \displaystyle\int_0^8 y(x)\,dx$

3 a) $\int \underbrace{(\sin t)^2}_{u^2} \underbrace{\cos t\, dt}_{du}$

Let $u = \sin t$, then $du = \cos t\, dt$

$= \int u^2\, du = \frac{1}{3}u^3 = \boxed{\frac{1}{3}(\sin t)^3 + C}$

b) $\int \frac{5^{\sqrt{t}}}{\sqrt{t}}\, dt$ $\leftarrow 5^u$, $2\,du$

Let $u = \sqrt{t}$, then $du = \frac{1}{2\sqrt{t}}\, dt$

$\left[(t^{1/2})' = \frac{1}{2} t^{-\frac{1}{2}} \right]$

$= 2\int 5^u\, du = 2 \cdot \frac{1}{\ln 5} 5^u = \boxed{\frac{2}{\ln 5} 5^{\sqrt{t}} + C}$

c) $\int \underbrace{t}_{v}\, \underbrace{3^{2t-1}}_{u'}\, dt$

$\int u'v = uv - \int uv'$

$v' = 1$, $u = \int u' = \int 3^{2t-1}\, dt =$ $\leftarrow 2^w$ $\leftarrow \frac{1}{3} dw$

Let $w = 2t-1$ then, $dw = 2\, dt$

$= \frac{1}{2}\int 3^w\, dw = \frac{1}{2\ln 3} 3^w = \frac{1}{2\ln 3} 3^{2t-1}$

$= \frac{t}{2\ln 3} 3^{2t-1} - \frac{1}{2\ln 3}\int 3^{2t-1}\, dt$

$\boxed{= \frac{t}{2\ln 3} 3^{2t-1} - \frac{1}{(2\ln 3)^2} 3^{2t-1} + C}$ $= \frac{3^{2t-1}}{2\ln 3}\left[t - \frac{1}{2\ln 3} \right] + C$

Shortcut:

differentiate ↓		integrate ↓
t		3^{2t-1}
1	$+$	$\frac{1}{2\ln 3} 3^{2t-1}$
0	$-$	$\frac{1}{(2\ln 3)^2} 3^{2t-1}$