MAT 1193.003 Midterm 1 Fall 2014

(1) Let $b(t)$ be the size of the colony (in millions) at time $t$ (days).

$b(3) = 8$, $b(3+4) = b(7) = 12$. $b(7+2) = b(9) = ?$

Exponential model: $b(t) = b(0)\,a^t$ for some $a > 0$.
(rate: $r = \ln a$)

$8 = b(3) = b(0)\,a^3$
$12 = b(7) = b(0)\,a^7$

divide: $\frac{2}{3}\ \frac{\cancel{8}}{\cancel{12}} = \frac{\cancel{b(0)}\,a^3}{\cancel{b(0)}\,a^7}$

$\frac{a^7}{a^3} = a^{7-3} = a^4 = \frac{3}{2}$ $\therefore$ $a = \sqrt[4]{\frac{3}{2}} \approx 1.0668192$

$[b(0) = 2^{15/4} \cdot 3^{-3/4},\ b(9) = 6^{3/2}]$

$b(0) = 8 \cdot a^{-3} \approx 5.90$

$b(9) = b(0)\,a^9 \approx 14.6969$

$\therefore$ After 9 days Justin Bieber will have 14.6969 million bacteria.

(2) a) Let $\Delta t = 10^{-3}$ $y'(5) \approx \frac{\Delta y}{\Delta t} = \frac{y(5+\Delta t) - y(5)}{\Delta t} =$

$= [y(5.001) - y(5)]\,10^3 = 6 \cdot 10^3 [2^{5.001} - 2^5] \approx 133.130$

With $\Delta t = -10^{-3}$ $y'(5) \approx -6 \cdot 10^3 [2^{4.999} - 2^5] \approx 133.038$

$\therefore$ $y'(5)$ is between 133.038 and 133.130

$[y'(5) = 6 \cdot \ln 2 \cdot 2^5 \approx 133.084]$

b) DNA grows at the rate of approximately 133 ng/min.

③

| t | 0 | 1.8 | 2.8 | 4 | 5 |
|---|---|---|---|---|---|
| slope | -3.5 | 0 | 1 | 0 | -2.4 |

| t | $0 \le t < 3$ | $3 < t \le 5$ |
|---|---|---|
| slope | $4/3$ | $-3/2$ |