(1) a) $y = \dfrac{\sin(3x)}{x^2}$

$$y' = \frac{\cos(3x)\cdot 3\cdot x^2 - \sin(3x)\cdot 2x}{x^4}$$

b) $y = x^{x^3}$ $\quad \ln y = \ln(x^{x^3}) = x^3 \ln x$

$$\frac{1}{y}\cdot y' = 3x^2 \ln x + x^3 \frac{1}{x} = x^2(3\ln x + 1)$$

$$y' = y\, x^2(3\ln x + 1) = x^{x^3} x^2 (3\ln x + 1)$$

$$= x^{x^3+2}(3\ln x + 1)$$

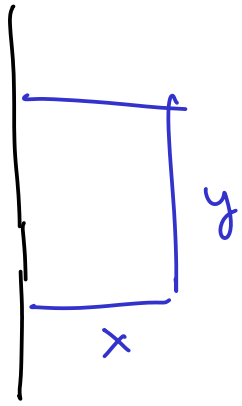
c) $\sin(5y) + \exp(4y) = x^3$

$$\cos(5y)\,5y' + \exp(4y)\,4y' = 3x^2$$

$$y'(\cos(5y)5 + \exp(4y)4) = 3x^2$$

$$y' = \frac{3x^2}{\cos(5y)5 + \exp(4y)4}$$

(2)



Objective: $A = xy$

Constraint: $2x + y = 30$

$$y = 30 - 2x$$

$$A = x(30 - 2x) = 30x - 2x^2$$

$$\frac{dA}{dx} = 30 - 4x \qquad \frac{dA}{dx} = 0 \Rightarrow x = \frac{30}{4} = 7.5\,m$$

$$y = 30 - 2 \cdot 7.5 = 15\,m$$

$$A = 15 \cdot 7.5 = 112.5\,m^2$$

(3)

a) $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$ $\left(\frac{0}{0}\right)$

$$\frac{e^x}{\cos x} \to 1$$

b) $\lim_{x \to 0^+} x^2 \ln x$ $\qquad \frac{\ln x}{1/x^2}$ $\left(\frac{\infty}{\infty}\right)$

$$\frac{\frac{1}{x}}{-\frac{2}{x^3}} = -\frac{1}{2}x^2 \to 0$$

c) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right)$ $\qquad \frac{x^{1/2} - x}{x^{3/2}}$ $\left(\frac{0}{0}\right)$

$x^{1/2}$

$$\frac{\frac{1}{2}x^{-\frac{1}{2}} - 1}{\frac{3}{2}x^{1/2}} = \frac{\frac{1}{2\sqrt{x}} - 1}{\frac{3}{2}\sqrt{x}} \to \frac{\infty}{0} = \infty$$

④ a) $\int \frac{1}{1+25x^2}\,dx = \int \frac{1}{1+(5x)^2}\,dx$

let $u = 5x$ $\quad \frac{du}{dx} = 5$ $\quad dx = \frac{1}{5}du$

$\frac{1}{5}\int \frac{1}{1+u^2}\,du = \frac{1}{5}\arctan u = \frac{1}{5}\arctan(5x) + C$

b) $\int x\sin(3x^2)\,dx$

let $u = 3x^2$ $\quad \frac{du}{dx} = 6x$ $\quad x\,dx = \frac{1}{6}du$

$\frac{1}{6}\int \sin u\,du = -\frac{1}{6}\cos u = -\frac{1}{6}\cos(3x^2) + C$

c) $\int x^2\sin(3x)\,dx = -\frac{x^2}{3}\cos(3x) + \frac{2x}{9}\sin(3x) +$

$+ \frac{2}{27}\cos(3x) + C$

x^2		$\sin(3x)$
$2x$	$+$	$-\frac{1}{3}\cos(3x)$
2	$-$	$-\frac{1}{9}\sin(3x)$
0	$+$	$\frac{1}{27}\cos(3x)$

(5) Let $y(t)$ = amt. of drug at time t

$$\frac{dy}{dt} = 1.5 - (0.7 - 0.1t^3)$$

$$= 0.8 + 0.1t^3 \qquad y(0) = 25$$

$$y = 0.8t + 0.1\frac{t^4}{4} = 0.8t + 0.025t^4 + C$$

Plug in $t=0$, $y=25$: $\quad 25 = 0 + 0 + C$

$$\therefore C = 25$$

$$\therefore y = 0.8t + 0.025t^4 + 25$$

$$y(4) = 34.6 \text{ mg}$$