

①

$$b(t) = b_0 e^{rt}$$

$$b(1) = 6 = b_0 e^r$$

$$b(2) = 7 = b_0 e^{r \cdot 2} = b_0 (e^r)^2$$

$$e^r = \frac{6}{b_0}$$

$$7 = b_0 \left(\frac{6}{b_0}\right)^2$$

$$7 = \frac{6^2}{b_0}, \quad b_0 = \frac{6^2}{7}$$

$$6 = \frac{6^2}{7} e^r$$

$$e^r = \frac{7}{6}$$

$$r = \ln\left(\frac{7}{6}\right)$$

$$\approx 5.143$$

$$\approx 0.15415$$

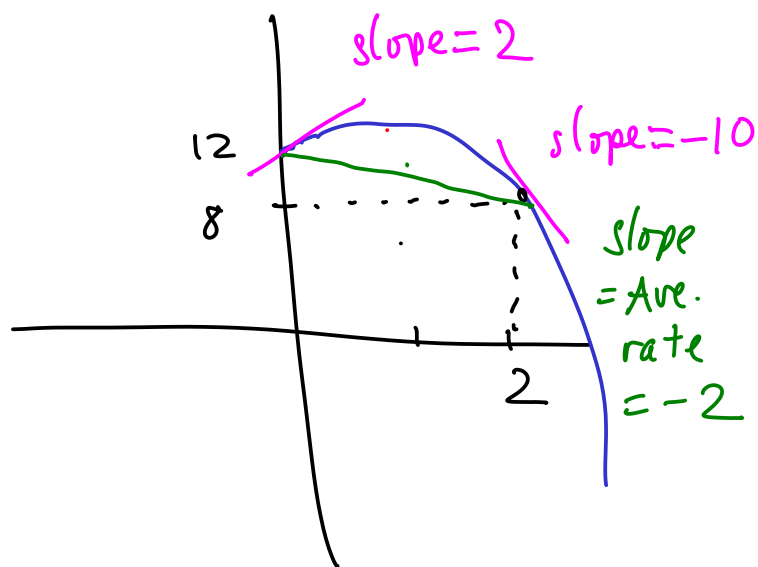
$$b(t) = 5.143 e^{0.15415 \cdot t}$$

$$b(3) = 5.143 e^{0.15415 \cdot 3} \approx \boxed{8.2 \text{ mil}}$$

$$(2) \quad S(t) = 12 + 2t - t^3$$

$$a) \quad S(0) = 12$$

$$S(2) = 8$$



$$S'(t) = \lim_{h \rightarrow 0} \frac{S(t+h) - S(t)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{12} + 2(t+h) - (t+h)^3 - (\cancel{12} + 2t - t^3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2t} + \cancel{2t} + 2h - \cancel{t^3} - 3t^2h - 3th^2 - h^3 - \cancel{2t} + \cancel{t^3}}{h}$$

$$= \lim_{h \rightarrow 0} (2 - 3t^2 - 3th - h^2) = \boxed{2 - 3t^2}$$

$$b) \quad S'(0) = 2, \quad S'(2) = -10$$

$$c) \quad \text{Ave. rate: } \frac{\Delta S}{\Delta t} = \frac{8-12}{2-0} = \boxed{-2}$$

(3)

$$f(x) = 20 - 2x^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cancel{20} - 2(x+h)^2 - (\cancel{20} - 2x^2)}{h}$$

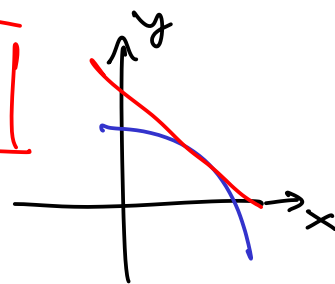
$$= \lim_{h \rightarrow 0} \frac{-\cancel{2x^2} - 4xh - 2h^2 + \cancel{2x^2}}{h}$$

$$= \lim_{h \rightarrow 0} (-4x - 2h) = \boxed{-4x}$$

$$y(x+h) = y(x) + y'(x)h + \epsilon$$

$$y(\underbrace{2+h}_x) = y(2) + y'(2)h = 12 - 8 \cdot h$$

$$y = 12 - 8(x-2) = \boxed{28 - 8x}$$



$$\textcircled{4} \quad a) \quad \lim_{n \rightarrow \infty} \frac{n}{5n+1} = \lim_{n \rightarrow \infty} \frac{1}{5 + \frac{1}{n}}$$

$$= \frac{1}{5 + \lim_{n \rightarrow \infty} \frac{1}{n}} = \frac{1}{5 + 0} = \frac{1}{5}$$

$$b) \quad \lim_{x \rightarrow 1} \frac{1-x}{1-x^2} = \lim_{x \rightarrow 1} \frac{\cancel{1-x}}{\cancel{(1-x)}(1+x)}$$

$$= \lim_{x \rightarrow 1} \frac{1}{1+x} = \frac{1}{2}$$

$$c) \quad \lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

Sandwich:  $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$



$$d) \quad \lim_{x \rightarrow 0} \frac{x}{\sin(3x)} =$$

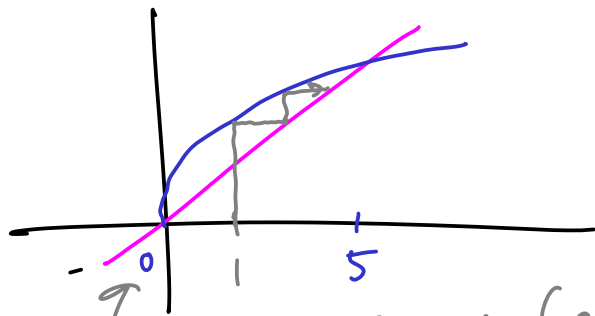
$$= \lim_{x \rightarrow 0} \frac{3x}{\sin(3x)} \cdot \frac{1}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

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$$x_{t+1} = \sqrt{5x_t}$$

Fixed pts:  $x = \sqrt{5} \sqrt{x}$

$$x=0 \quad \text{or} \quad \sqrt{x} = \sqrt{5}, \quad x=5$$



unstable (repelling)      stable (attracting)

If  $x_0 = 0$ ,  $x_n = 0$  for all  $n$

If  $x_0 = 1$ ,  $x_n \rightarrow 5$  as  $n \rightarrow \infty$