(1) a)

$$y = \frac{\cos(5x)}{x^2}$$

$$y' = \frac{-\sin(5x)\cdot 5\cdot x^2 - \cos(5x)\cdot 2x}{x^4}$$

b)

$$y = x^{x^2} \qquad \ln y = \ln\left(x^{x^2}\right)$$

$$= x^2 \ln x$$

$$\frac{1}{y} y' = 2x\cdot \ln x + x^2 \frac{1}{x} = x\,(2\ln x + 1)$$

$$y' = y\,x\,(2\ln x + 1) = x^{x^2}\cdot x\,(2\ln x + 1)$$

$$= x^{x^2+1}\,(2\ln x + 1)$$

c)

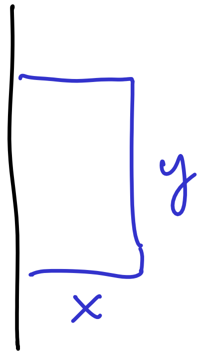
$$\cos(3y) + \exp(2y) = x^2$$

$$-\sin(3y)\,3y' + \exp(2y)\,2y' = 2x$$

$$y'\left(-\sin(3y)3 + \exp(2y)2\right) = 2x$$

$$y' = \frac{2x}{-\sin(3y)3 + \exp(2y)2}$$

(2)



Objective : $A = xy$

Constraint : $2x + y = 20$

$$y = 20 - 2x$$

$$A = x(20 - 2x) = 20x - 2x^2$$

$$\frac{dA}{dx} = 20 - 4x \qquad \frac{dA}{dx} = 0 \Rightarrow \underline{x = 5\ m}$$

$$y = 20 - 2 \cdot 5 = \underline{10\ m}$$

$$A = 10 \cdot 5 = \underline{50\ m^2}$$

③ a) $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$ $\left(\frac{0}{0}\right)$

$$\frac{e^x}{\cos x} \to 1$$

b) $\lim_{x \to 0^+} x^2 \ln x$ $\qquad \frac{\ln x}{\frac{1}{x^2}}$ $\left(\frac{\infty}{\infty}\right)$

$$\frac{\frac{1}{x}}{-\frac{2}{x^3}} = -\frac{1}{2}x^2 \to 0$$

c) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) \to \frac{x^{\frac{1}{2}} - x}{x^{3/2}}$ $\left(\frac{0}{0}\right)$

$$\frac{\frac{1}{2}x^{-\frac{1}{2}} - 1}{\frac{3}{2}x^{\frac{1}{2}}} = \frac{\frac{1}{2\sqrt{x}} - 1}{\frac{3}{2}\sqrt{x}} \to \frac{\infty}{0} = \infty$$

④ a) $\int \frac{1}{1+9x^2}\,dx = \int \frac{1}{1+(3x)^2}\,dx$

Let $u = 3x$ $\quad \frac{du}{dx} = 3 \quad dx = \frac{1}{3}\,du$

$\frac{1}{3}\int \frac{1}{1+u^2}\,du = \frac{1}{3}\arctan u = \frac{1}{3}\arctan(3x) + C$

b) $\int x\cos(2x^2)\,dx$

Let $u = 2x^2 \quad \frac{du}{dx} = 4x \quad x\,dx = \frac{1}{4}\,du$

$\frac{1}{4}\int \cos(u)\,du = \frac{1}{4}\sin(u) = \frac{1}{4}\sin(2x^2) + C$

c) $\int x^2\cos(2x)\,dx = \frac{x^2}{2}\sin(2x) + \frac{x}{2}\cos(2x) -$
$- \frac{1}{4}\sin(2x) + C$

$x^2 \qquad \cos(2x)$

$2x \quad + \quad \frac{1}{2}\sin(2x)$

$2 \quad - \quad -\frac{1}{4}\cos(2x)$

$0 \quad + \quad -\frac{1}{8}\sin(2x)$

⑤ Let $y(t)$ = amt. of drug at time t.

$$\frac{dy}{dt} = 1.2 - (0.8 - 0.1t^2)$$
$$= 0.4 + 0.1t^2$$

Initial cond: $y(0) = 15$

$$y = 0.4t + 0.1\frac{t^3}{3} + C$$

Plug in: $t = 0$, $y = 15$

$$15 = 0 + 0 + C \qquad \therefore C = 15$$

$$\therefore \quad y(t) = 0.4t + \frac{0.1}{3}t^3 + 15$$

$$y(6) = 24.6 \text{ mg}$$