MAT 1193.006-007 Final Exam Fall 2012

(1) Let $b(t)$ denote the size of the culture at time $t$.

Then $b(t) = b_0\, a^t$, where $b_0$ is the initial size and $a > 1$.

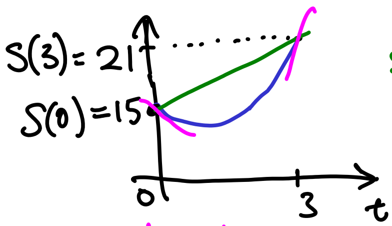
006: $4 = b_0\, a^2$, $6 = b_0\, a^3$. Divide: $a = \frac{6}{4} = \frac{3}{2}$

$4 = b_0\left(\frac{3}{2}\right)^2$, $b_0 = \frac{2^4}{3^2}$, $b(4) = \frac{2^4}{3^2}\left(\frac{3}{2}\right)^4 = 3^2 = \underline{9 \text{ mil}}$

007: $5 = b_0\, a^3$, $7 = b_0\, a^4$, Divide: $a = \frac{7}{5}$.

$5 = b_0\left(\frac{7}{5}\right)^3$, $b_0 = \frac{5^4}{7^3}$, $b(5) = \frac{5^4}{7^3}\left(\frac{7}{5}\right)^5 = \frac{7^2}{5} = \underline{9.8 \text{ mil}}$

(2) $s(t) = 15 + t^2 - t$




slope = ave. rate = $\frac{21-15}{3-0} = 2$

$$s'(t) = \lim_{h\to 0} \frac{\cancel{15} + (t+h)^2 - (\cancel{t}+h) - (\cancel{15} + t^2 - \cancel{t})}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{t^2} + 2th + h^2 - h - \cancel{t^2}}{h}$$

$$= \lim_{h\to 0} 2t + h - 1 = 2t - 1$$

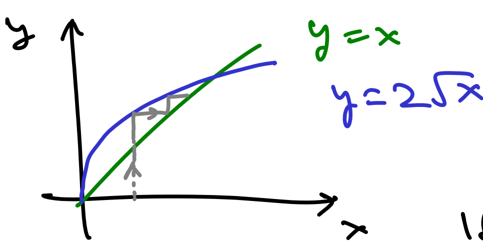
slopes = inst. rates:

$s'(0) = -1$, $s'(3) = 5$

Tangent lines:

$y = 15 - t$, $y = 21 + 5(t-3)$

(3) 006: $x_{t+1} = 2\sqrt{x_t}$




Fixed points: $x = 2\sqrt{x}$

$\Rightarrow \sqrt{x} = 0$ or $\sqrt{x} = 2$

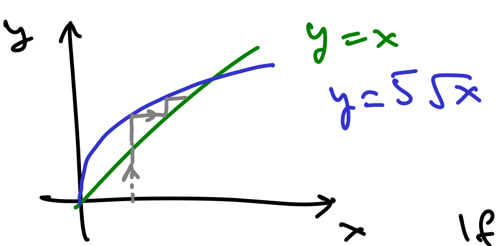
$\Rightarrow \underline{x = 0}$ or $\underline{x = 4}$

repelling attracting

If $x_0 = 0$, then for all $t$ $x_t = 0$

If $x_0 > 0$, then $\lim_{t\to\infty} x_t = 4$

007: $x_{t+1} = 5\sqrt{x_t}$




Fixed points: $x = 5\sqrt{x}$

$\Rightarrow \sqrt{x} = 0$ or $\sqrt{x} = 5$

$\Rightarrow \underline{x = 0}$ or $\underline{x = 25}$

repelling attracting

If $x_0 = 0$, then for all $t$ $x_t = 0$

If $x_0 > 0$, then $\lim_{t\to\infty} x_t = 25$

④ a) 006: $y = x^3 \ln x$, $y' = 3x^2 \ln x + x^{\cancel{3}\,2} \frac{1}{\cancel{x}} = x^2(3\ln x + 1)$

007: $y = x^4 \ln x$, $y' = 4x^3 \ln x + x^{\cancel{4}\,3} \frac{1}{\cancel{x}} = x^3(4\ln x + 1)$

b) 006: $y = \dfrac{x^2}{\cos(5x)}$, $y' = \dfrac{2x\cos(5x) + x^2 \cdot \sin(5x)5}{[\cos(5x)]^2}$

007: $y = \dfrac{x^3}{\sin(2x)}$, $y' = \dfrac{3x^2\sin(2x) - x^3\cos(2x)2}{[\sin(2x)]^2}$

c) $y = x^{e^x}$, $\ln y = \ln(x^{e^x}) = e^x \ln x$

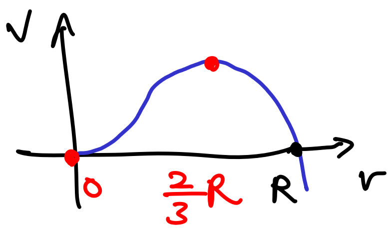
$\dfrac{y'}{y} = e^x \ln x + e^x \dfrac{1}{x}$, $y' = x^{e^x} e^x\left(\ln x + \dfrac{1}{x}\right)$

d) $\sin(3y) + \exp(2x) = y^2$, $\cos(3y)3y' + \exp(2x)2 = 2yy'$

$y' = \dfrac{2\exp(2x)}{2y - 3\cos(3y)}$

⑤ $V = a(R-r)r^2 = a(Rr^2 - r^3)$, $\dfrac{dV}{dr} = a(2Rr - 3r^2)$

$= ar(2R - 3r)$




$\therefore \dfrac{dV}{dr} = 0 \Rightarrow r = 0$ or $r = \dfrac{2R}{3}$

Max V occurs when $r = \frac{2}{3}R$.

⑥ a) 006: $\lim\limits_{x\to\infty} \dfrac{x}{4x+1} = \lim\limits_{x\to\infty} \dfrac{1}{4 + \frac{1}{x}} = \dfrac{1}{4}$

007: $\lim\limits_{x\to\infty} \dfrac{x}{3x+1} = \lim\limits_{x\to\infty} \dfrac{1}{3 + \frac{1}{x}} = \dfrac{1}{3}$

b) 006: $-1 \le \cos\left(\frac{1}{x}\right) \le 1$, $-x^4 \le x^4\cos\left(\frac{1}{x}\right) \le x^4$

$\to 0 \leftarrow$

007: $-1 \le \sin\left(\frac{1}{x}\right) \le 1$, $-x^4 \le x^4\sin\left(\frac{1}{x}\right) \le x^4$

$\to 0 \leftarrow$

c) 006: $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{3x} = \dfrac{5}{3}\lim\limits_{x\to 0}\dfrac{\sin(5x)}{5x} = \dfrac{5}{3}$

007: $\lim\limits_{x\to 0} \dfrac{\sin(3x)}{5x} = \dfrac{3}{5}\lim\limits_{x\to 0}\dfrac{\sin(3x)}{3x} = \dfrac{3}{5}$

d) 006: $x^3 \ln x = \dfrac{\ln x}{\frac{1}{x^3}}$ $\left(\frac{\infty}{\infty}\right)$ l'Hôpital's Rule:

$$\frac{\frac{1}{x}}{-\frac{3}{x^4}} = -\frac{1}{3}x^3 \to 0 \text{ as } x \to 0^+$$

007: $x^4 \ln x = \dfrac{\ln x}{\frac{1}{x^4}}$ $\left(\frac{\infty}{\infty}\right)$ l'Hôpital's Rule:

$$\frac{\frac{1}{x}}{-\frac{4}{x^5}} = -\frac{1}{4}x^4 \to 0 \text{ as } x \to 0^+$$

7. a) $\int \sin(3x)\,dx = -\frac{1}{3}\cos(3x) + C$

b) $\int x \sin(3x^2)\,dx = -\frac{1}{6}\cos(3x^2) + C$

(Let $u = 3x^2$ then $\frac{du}{dx} = 6x$, so $x\,dx = \frac{1}{6}du$)

c) $\int x^2 \sin(3x)\,dx = -\frac{x^2}{3}\cos(3x) + \frac{2x}{9}\sin(3x) +$

$+ \frac{2}{27}\cos(3x) + C$

| | | |
|---|---|---|
| $x^2$ | | $\sin(3x)$ |
| | $+$ | |
| $2x$ | | $-\frac{1}{3}\cos(3x)$ |
| | $-$ | |
| $2$ | | $-\frac{1}{9}\sin(3x)$ |
| | $+$ | |
| $0$ | | $\frac{1}{27}\cos(3x)$ |

8. 006: $y' = 1.4 - (0.9 - 0.05t^4) = 0.5 + 0.05t^4$ $y(0) = 25$

$y = 0.5t + 0.01t^5 + 25$, $y(3) = \underline{28.93 \text{ mg}}$

007: $y' = 1.2 - (0.8 - 0.05t^3) = 0.4 + 0.05t^3$ $y(0) = 22$

$y = 0.4t + 0.05\frac{t^4}{4} + 22$, $y(4) = \underline{26.8 \text{ mg}}$

Have a great break!