Final exam MAT 1193.005 Fall 2011

#1

```
> p:=p0*a^t;
```

$$p := p0\ a^t$$

```
> {subs(t=3,p)=25,subs(t=4,p)=30}; sol:=solve(%,{a,p0}); evalf(%);
```

$$\left\{p0\ a^3 = 25, p0\ a^4 = 30\right\}$$

$$sol := \left\{a = 6/5, p0 = \tfrac{3125}{216}\right\}$$

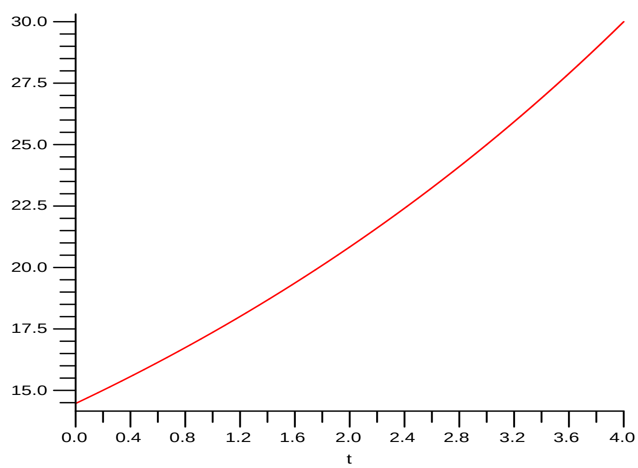
$$\{a = 1.200000000, p0 = 14.46759259\}$$

```
> evalf(ln(1.2));
```

$$0.1823215568$$

```
> subs(sol,p); plot(%,t=0..4);
```

$$\tfrac{3125}{216}\ (6/5)^t$$



Initial value is about 14.5 million.

#2

```
> t^2*7^(3^t); diff(%,t);
```

$$t^2 7^{3^t}$$

$$2\,t 7^{3^t} + t^2 7^{3^t} 3^t \ln(3) \ln(7)$$

```
> ln(t)/sqrt(2*t^3); diff(%,t);
```

$$1/2\,\frac{\ln(t)\sqrt{2}}{\sqrt{t^3}}$$

$$1/2\,\frac{\sqrt{2}}{t\sqrt{t^3}} - 3/4\,\frac{\ln(t)\sqrt{2}t^2}{(t^3)^{3/2}}$$

```
> sin(t)*cos(t); diff(%,t);
```

$$\sin(t)\cos(t)$$

$$(\cos(t))^2 - (\sin(t))^2$$

#3-4 (see midterm)

#5

Take the derivative and find where it is 0

```
> f:=sin; df:=diff(f(x),x); solve(%,x);
```

$$f := \sin$$

$$df := \cos(x)$$

$$1/2\,\pi$$

Maple missed one solution, but the critical points in the interval are pi/2, 3pi/2

```
> diff(df,x); subs(x=Pi/2,%); eval(%);
> diff(df,x); subs(x=3*Pi/2,%); eval(%);
```

$$-\sin(x)$$

$$-\sin(1/2\,\pi)$$

$$-1$$

$$-\sin(x)$$

$$-\sin(3/2\,\pi)$$

$$1$$

So pi/2 is a local max and 3pi/2 a local min.

Now collect critical and end points and find their values:

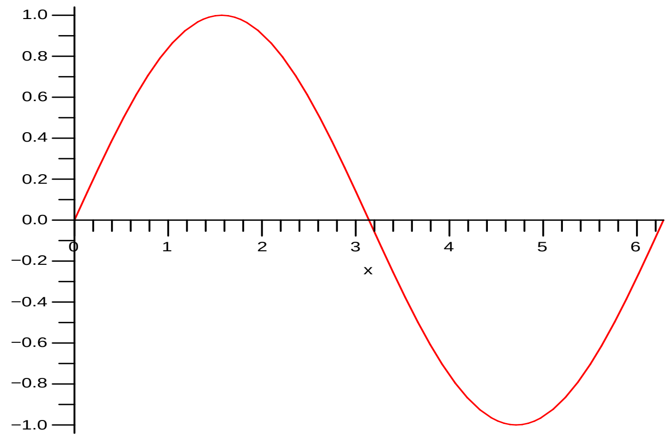
```
> [0,2*Pi,Pi/2,3*Pi/2]; map(w->f(w),%);
```

$$[0, 2\,\pi, 1/2\,\pi, 3/2\,\pi]$$

$$[0, 0, 1, -1]$$

Thus, global max is 1. It occurs at x=pi/2. Global min is -1. It occurs at x=3pi/2.

```
> plot(f(x),x=0..2*Pi);
```



#6

```
> exp(3*x)/(1-5*exp(3*x))^2; int(%,x);
```

$$\frac{e^{3\,x}}{(1-5\,e^{3\,x})^2}$$

$$1/15\,\left(1-5\,e^{3\,x}\right)^{-1}$$

```
> 1/(x*ln(x)^2); int(%,x);
```

$$\frac{1}{x(\ln(x))^2}$$

$$-\left(\ln\left(x\right)\right)^{-1}$$

```
> t^3*exp(-5*t); int(%,t); expand(%);
```

$$t^3e^{-5\,t}$$

$$-\frac{1}{625}\,\left(6+30\,t+75\,t^2+125\,t^3\right)e^{-5\,t}$$

$$-\frac{6}{625}\,\left(e^t\right)^{-5}-\frac{6}{125}\,\frac{t}{(e^t)^5}-\frac{3}{25}\,\frac{t^2}{(e^t)^5}-1/5\,\frac{t^3}{(e^t)^5}$$

#7

The given integral is less than the integral of x^(-6/5), which converges, since 6/5 is greater than 1.

```
> 1/(x^(4/5)+x^(6/5)); %/x^(-6/5); limit(%,x=infinity);
```

$$\left(x^{4/5}+x^{6/5}\right)^{-1}$$

$$\frac{x^{6/5}}{x^{4/5}+x^{6/5}}$$

$$1$$

```
> int(x^(-6/5),x=1..infinity);
```

#8

```
> f:=x-a*x^2;
```

$$f := x - ax^2$$

```
> equi:=[solve(f,x)];
```

$$equi := [0, a^{-1}]$$

```
> df:=diff(f,x);
```

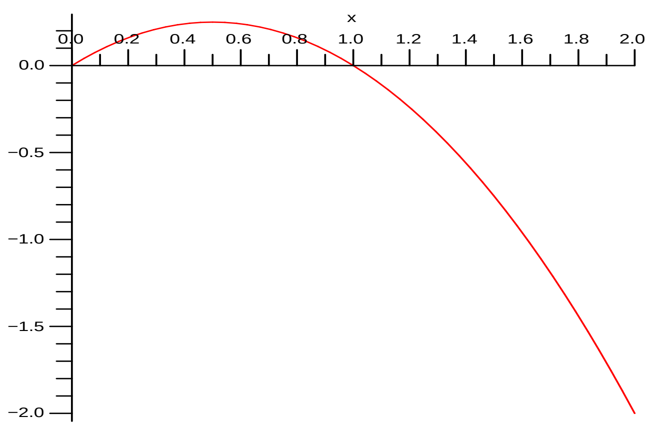
$$df := 1 - 2\,ax$$

```
> map(w->subs(x=w,df),equi);
```

$$[1, -1]$$

Thus, 0 is unstable and 1/a stable.

```
> plot(subs(a=1,f),x=0..2);
```



It turns out this equation can be solved exactly. Just for fun, here are some solutions.

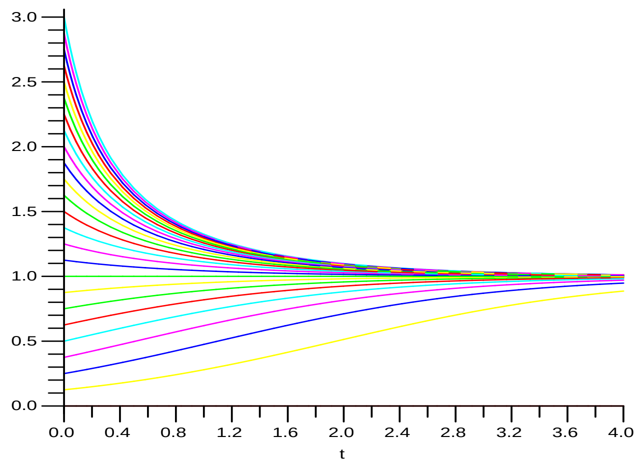
```
> eq:=diff(x(t),t)=subs(x=x(t),f);
```

$$eq := \frac{d}{dt}x(t) = x(t) - a(x(t))^2$$

```
> dsolve({eq,x(0)=1},x(t));
```

$$x(t) = -\left(-a - e^{-t} + e^{-t}a\right)^{-1}$$

```
> [seq(k*3*a/24,k=0..24)]:
> map(v->subs(dsolve({x(0)=v,eq},x(t)),x(t)),%):
> subs(a=1,%): plot(convert(%,set),t=0..4);
```



#9

```
> eq:=diff(h(t),t)=-h(t)^2; ic:=h(0)=2;
```

$$eq := \frac{d}{dt}h(t) = -(h(t))^2$$

$$ic := h(0) = 2$$

```
> dsolve({eq,ic},h(t)); hh:=subs(%,h(t)):
```

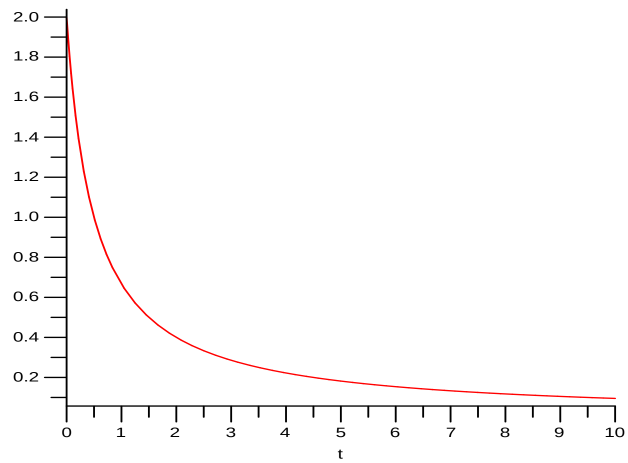
$$h(t) = 2\,(1 + 2\,t)^{-1}$$

```
> limit(hh,t=infinity);
```

$$0$$

Long term behaviour: medium-fast decay to zero.

```
> plot(hh,t=0..10);
```


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
t

>