Midterm 2 / 2001.11.28 / Precalculus (Honors) / MAT 1093.002

Name: _____

Please show all work and provide clear explanations. Sketch. Include units. Make conclusions.

1. (20 pts.) Daisy Mae measures the yearly yield of her orchard (in dozens of ugli fruit per tree) for a few years and succeeds at finding a quadratic model $Y(t) = 1 + 0.3t - 0.01t^2$ that closely matches her data.

   (a) Find the vertex form of $Y(t)$ and deduce the model's prediction for the time when the yield reaches maximum?

   (b) What will be the maximum yield?

2. (20 pts.) Daisy Mae is riding on a ferris wheel built on a river bank. The diameter of the wheel is 20 meters. The wheel's axis is parallel to the river. The axis rests on a vertical column which is 8 meters away from the water. What fraction of the time is Daisy Mae above water?

3. (20 pts.) Biff's truck is getting old and he is going to need $20,000 five years from now to buy a new one. He decides to buy a Certificate of Deposit bearing 5% APR interest compounded quarterly.

   (a) How much money does Biff need to deposit?

   (b) What if the interest is compounded continuously?

4. (20 pts.) After Biff has an espresso, he metabolizes 7% of the caffeine each minute.

   (a) What is the half-life of caffeine in Biff's body?

   (b) Daisy Mae's metabolism is different from Biff's and the half-life of caffeine in her body is 5 minutes. What is the percentage of caffeine metabolized by Daisy Mae each minute?

1	2	3	4	total (80)	(%)

① $Y(t) = 1 + 0.3t - 0.01t^2$

$= -0.01(t^2 - 30t - 100)$

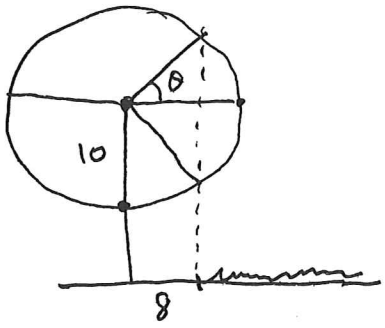
$= -0.01(t^2 - 2 \cdot 15t + 15^2 - 15^2 - 100)$

$= -0.01(t-15)^2 + 3.25$

Max yield = 3.25 dozen ugli fruit per tree

Max. occurs 15 years down the line

②



$\cos\theta = \frac{8}{10} = 0.8$

$\theta = \arccos(0.8) = 0.6435$

$\frac{\theta}{\pi} = 0.2$

So Daisy Mae is above water 20% of the time.

(3) a) Let P_0 denote the initial deposit.

$$P(t) = P_0\left(1+\frac{r}{n}\right)^{nt} = P_0\left(1+\frac{0.5}{4}\right)^{4t}$$

(t ↑ years)

Biff wants: $20000 = P_0\left(1+\frac{.05}{4}\right)^{4\cdot 5}$

Solve for P_0:

$$P_0 = 20000\left(1+\frac{.05}{4}\right)^{-4\cdot 5} = \boxed{15600\ \$}$$

b) $P(t) = P_0 e^{rt} = P_0 e^{.05t}$

Biff Wants: $20000 = P_0 e^{.05\cdot 5}$

Solve for P_0:

$$P_0 = 20000\, e^{-.05\cdot 5} = \boxed{15576\ \$}$$

(4) a) Rate of decay: $r = -0.07$

Base: $b = 1 + (-0.07) = 0.93$

Amount of caffeine in Biff's body:

$$Q(t) = Q_0 b^t = Q_0 \cdot 0.93^t$$

(t ↑ minutes)

To find half-life solve for t:

$$\frac{Q_0}{2} = Q_0\, 0.93^t$$

$$\frac{1}{2} = 0.93^t$$

Take logs:

$$\ln\left(\tfrac{1}{2}\right) = t \ln(0.93)$$

$$t = \frac{\ln(0.5)}{\ln(0.93)} = 9.55 \text{ minutes}$$

b) For Daisy Mae:

$$\frac{1}{2} = b^5 \quad \text{, solve for } b\text{:}$$

$$b = \left(\tfrac{1}{2}\right)^{1/5} = .87$$

$b = 1 + r$ solve for r:

$$r = b - 1 = .87 - 1 = -.13$$

Daisy Mae metabolizes 13% of caffeine per minute