CS 3333.002 Mid 3 Sp'16

① Laplace expansion along the last row

$$\det = -7 \det \begin{bmatrix} 3 & 0 & 2 \\ 3 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} + 4 \det \begin{bmatrix} 2 & 3 & 0 \\ 4 & 3 & 2 \\ 6 & 0 & 0 \end{bmatrix}$$

$$= -7 \cdot 3 \det \begin{bmatrix} 3 & 0 \\ 3 & 2 \end{bmatrix} + 4 \cdot 6 \det \begin{bmatrix} 3 & 0 \\ 3 & 2 \end{bmatrix} = (-21+24)6 = \boxed{18}$$

② 5 card draws: $C(52,5)$

In each suit: flushes: $C(13,5)$

straights: 10

$C(n,k) = \frac{n!}{k!(n-k)!}$

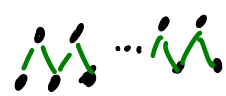
$$\frac{4[C(13,5)-10]}{C(52,5)} = \frac{4(1287-10)}{2598960} = \frac{1277}{649740} \boxed{\approx 0.0019654}$$

③ a) Physical proof: Let X be a set of size n. $C(n,k) = $ # of subsets of size k.

$\therefore C(n,0) + \ldots + C(n,n) = $ # of all subsets $= 2^n$

Induction: Basis: $C(1,0) + C(1,1) = 1 + 1 = 2$ ☺

By Pascal's triangle each entry in the row above contributes twice to the sum:



$\leftarrow 2^{n-1}$

$\leftarrow 2 \cdot 2^{n-1} = 2$

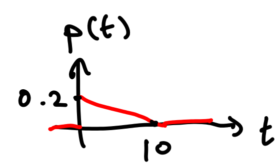
Binomial theorem: $(a+b)^n = C(n,0)a^n + C(n,1)a^{n-1}b + \ldots C(n,n)b^n$

$\therefore\ 2^n = (1+1)^n = C(n,0) + \ldots C(n,n)$

b) Induction: $C(1,0) - C(1,1) = 1 - 1 = 0$. The rest as above, but each pair of contributions cancels (opposite signs).

Binomial theorem: $0 = (1-1)^n = C(n,0) + C(n,1)(-1) + \ldots (-1)^n C(n,n)$

④ a) $\int_0^{10} (mt + 0.2)\,dt = \left[m\frac{t^2}{2} + 0.2t \right]_0^{10} = 50m + 2$



$50m + 2 = 1 \Rightarrow m = -\frac{1}{50} = \boxed{-0.02}$ $\therefore\ p(t) = 0.02(10-t)$

b) $\mu = \int_{-\infty}^{\infty} t\,p(t)\,dt = 0.02 \int_0^{10} (10t - t^2)\,dt = 0.02\left[5t^2 - \frac{t^3}{3}\right]_0^{10} = 2\left[5 - \frac{10}{3}\right] = \frac{10}{3} \boxed{\approx 3.33}$

c) $\mathrm{cdf}(x) = \int_{-\infty}^{x} p(t)\,dt = 0.02\int_{-\infty}^{x} (10-t)\,dt = 0.02\left[10t - \frac{t^2}{2}\right]_0^x = 0.02\left(10x - \frac{x^2}{2}\right)$

$\mathrm{cdf}(x) = \frac{1}{2} \Rightarrow 0.01x^2 - 0.2x + 0.5 = 0$, $x^2 - 20x + 50 = 0$, $x = 10 - 5\sqrt{2}$, $x \boxed{\approx 2.93}$

1

```
(%i1) A:matrix([2,3,0,2],[4,3,2,1],[6,0,0,3],[7,0,0,4]);
      determinant(A);
```

(%o1) $\begin{bmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 0 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{bmatrix}$

(%o2) 18

2

```
(%i3) C(n,k):=n!/k!/(n-k)!;
      f:4*(C(13,5)-10)/C(52,5);
      float(f);
```

(%o3) $C(n,k) := \frac{\frac{n!}{k!}}{(n-k)!}$

(%o4) $\frac{1277}{649740}$

(%o5) 0.0019654015452334

4

```
(%i6) p:m*t+0.2;
      integrate(p,t,0,10);
      solve(%=1,m);
      pp:substitute(%,p);
      integrate(t*pp,t,0,10);
      float(%);
      cdf:integrate(pp,t,0,x);
      solve(%=1/2,x);
      float(%);
```

(%o6) $m\,t+0.2$

rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 2.0 by 2/1 = 2.0

(%o7) $50\,m+2$

(%o8) $[m=-\frac{1}{50}]$

(%o9) $0.2-\frac{t}{50}$

rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.2 by 1/5 = 0.2

(%o10) $\frac{10}{3}$

(%o11) 3.333333333333333

rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced 0.2 by 1/5 = 0.2
rat: replaced 0.2 by 1/5 = 0.2

(%o12) $-\frac{x^2-20\,x}{100}$

(%o13) $[x=10-5\,\sqrt{2}, x=5\,\sqrt{2}+10]$

(%o14) $[x=2.928932188134524, x=17.07106781186547]$