(1) $9!$ (think of "42" as a single token)

(2) 8 with 5 : $n=5$, $k=8-5=3$

$$C(n+k-1,k) = C(5+3-1,3) =$$

$$= C(7,3) = \frac{7!}{4!\,3!} = \frac{7\cdot 6\cdot 5}{3!} = 7\cdot 5 = 35$$

$$P = \frac{35}{6^5} = \boxed{0.004}$$

$n=3$, $k=7-3=4$

$$C(n+k-1,k) = C(3+4-1,4)$$

$$= C(6,4) = \frac{6!}{2!\,4!} = \frac{6\cdot 5}{2} = 15$$

$$P = \frac{15}{6^3} = \boxed{0.0694}$$

(3) $p(E \cap F) = p(E)\,p(F) \Rightarrow p(\bar{E} \cap \bar{F}) = p(\bar{E})\,p(\bar{F})$

(really $\Leftrightarrow$ ☺)

<u>Proof:</u>

$$p(\bar{E} \cap \bar{F}) = p(\overline{E \cup F}) =$$

← de Morgan

$$= 1 - p(E \cup F)$$

← Inclusion/Exclusion

$$= 1 - [\,p(E) + p(F) - p(E \cap F)\,]$$

$$= 1 - p(E) - p(F) - p(E)\,p(F) =$$

$$= (1 - p(E))(1 - p(F)) =$$

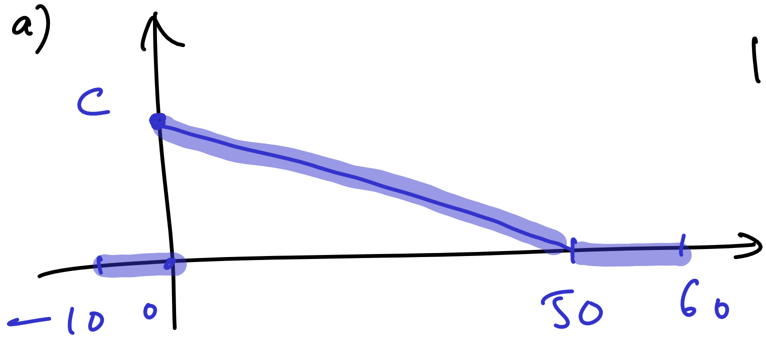
$$= p(\bar{E}) \cdot p(\bar{F})$$

☺

④

$$p(t) = \begin{cases} c(1-0.02\,t) & \text{for } 0 \le t \le 50 \\ 0 & \text{otherwise} \end{cases}$$

a)




$$1 = A = \frac{1}{2}\,50 \cdot c = 25c$$

$$\therefore\ c = \frac{1}{25} = \boxed{0.04}$$

b) $$P(t \le 10) = \int_0^{10} p(t)\,dt = 0.04\int_0^{10} (1-0.02t)\,dt$$

$$= 0.04\left[t - 0.02\frac{t^2}{2}\right]_0^{10} = \Big[\underbrace{0.04\,(t-0.01t^2)}_{\text{cdf}}\Big]_0^{10}$$

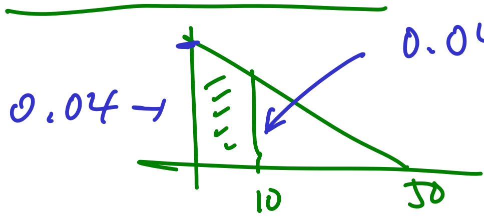
$$= \text{cdf}(10) = 0.04\,(10 - 0.01 \cdot 10^2)$$

$$= 0.04 \cdot 10\,(1-0.1) = 0.4 \cdot 0.9 = \boxed{0.36}$$

Check the cdf: $\text{cdf}(0) = 0$, $\text{cdf}(50)$

$$= 0.04\,(50 - 0.01\,50^2) = 2\,(1-0.5) = 1$$ ☺

Another check:




$$0.04(1 - 0.02 \cdot 10) = 0.04 \cdot 0.8$$

$$10 \cdot \frac{0.04 + 0.04 \cdot 0.8}{2} = 0.4\,\frac{1.8}{2} = 0.4 \cdot 0.9$$

☺

c) $\mu = \int_{-\infty}^{\infty} t\,p(t)\,dt = 0.04\int_0^{50} t(1-0.02t)\,dt$

$$= 0.04 \int_0^{50} (t - 0.02t^2)\,dt$$

$$= 0.04 \left[ \frac{t^2}{2} - 0.02\frac{t^3}{3} \right]_0^{50}$$

$$= 0.04 \left[ \frac{50^2}{2} - 0.02\frac{50^3}{3} \right] = \boxed{16.666\ldots}$$

d) Set $cdf(t) = \frac{1}{2}$, solve for $t$.

$$0.04(t - 0.01t^2) = 0.5$$

$$t - 0.01t^2 = \frac{0.5}{0.04} = 12.5$$

$$0.01t^2 - t + 12.5 = 0$$

$$t = \frac{1 \pm \sqrt{1 - 4 \cdot 0.01 \cdot 12.5}}{2 \cdot 0.01}$$

$$= \cancel{85.4},\ \boxed{14.64466}$$

(not in the interval)