1) $357 = 2 \cdot 178 + 1$

$178 = 2 \cdot 89 + 0$

$89 = 2 \cdot 44 + 1$

$44 = 2 \cdot 22 + 0$

$22 = 2 \cdot 11 + 0$

$11 = 2 \cdot 5 + 1$

$5 = 2 \cdot 2 + 1$

$2 = 2 \cdot 1 + 0$

$1 = 2 \cdot 0 + 1$

$357 = \boxed{(101100101)_2}$

$357 = 8 \cdot 44 + 5$

$44 = 8 \cdot 5 + 4$

$5 = 8 \cdot 0 + 5$

$\boxed{357 = (545)_8}$

$357 = 16 \cdot 22 + 5$

$22 = 16 \cdot 1 + 6$

$1 = 16 \cdot 0 + 1$

$\boxed{357 = 0x165}$

$0xFAB = 15 \times 16^2 + 10 \times 16 + 11$

$= 3840 + 160 + 11$

$\boxed{0xFAB = 4011}$

2) A hexadecimal number, d, can be represented in decimal form as $a_n \cdot 16^n + a_{n-1} \cdot 16^{n-1} + \ldots a_0 \cdot 16^0$.

$16 \bmod 5 = 1$

$16^2 \bmod 5 = 1$

$16^3 \bmod 5 = 1$

$16^4 \bmod 5 = 1$

From this we gather that $16^n \bmod 5 = 1$

So we can reduce the above representation:

$d = a_n + a_{n-1} + \ldots + a_0$, so $5 \mid d$ if and only if $(a_n + a_{n-1} + \ldots + a_0) \bmod 5 = 0$ (no remainder upon division by 5).

Thus d is only divisible by 5 if the sum of its digits in hexadecimal expansion is divisible by 5. $\square$

3) $252 = 1 \cdot 198 + 54$

$198 = 3 \cdot 54 + 36$

$54 = 1 \cdot 36 + 18$ ← last nonzero remainder

$36 = 2 \cdot 18 + 0$

$\boxed{\gcd(252, 198) = 18}$

$18 = 54 - 1 \cdot 36 = 54 - 1(198 - 3 \cdot 54)$

$= -1 \cdot 198 + 4 \cdot 54 = -1 \cdot 198 + 4(252 - 1 \cdot 198)$

$= 4 \cdot 252 - 5 \cdot 198$

$\boxed{\text{Bezout coefficients of } 252 + 198\text{: } 4, -5}$

4) $x \equiv 2 \bmod \overset{m_1}{5}$ $\qquad 6x \equiv 5 \bmod \overset{m_2}{7}$ $\qquad 7x \equiv 3 \bmod \overset{m_3}{8}$

$m = 5 \cdot 7 \cdot 8 = 280$

i	$M_i = m/m_i$	a_i	$y_i\ (M_i^{-1})$	$M_i a_i y_i$
1	$56 \equiv 1 \pmod 5$	2	1	$56 \cdot 2 \cdot 1 = 112$
2	$40 \equiv 5 \pmod 7$	2	3	$40 \cdot 2 \cdot 3 = 240$
3	$35 \equiv 3 \pmod 8$	5	3	$35 \cdot 5 \cdot 3 = 525$

$6x \equiv 1 \pmod 7$ $\qquad 7x \equiv 1 \bmod 8$

inverse of $6 \pmod 7$ is 6 $\qquad$ inverse of $7 \pmod 8$ is 7

$6x \equiv 5 \pmod 7$ $\qquad 7x \equiv 3 \bmod 8$

$6 \cdot 6x \equiv 5 \cdot 6 \pmod 7$ $\qquad 7 \cdot 7x \equiv 3 \cdot 7 \bmod 8$

$x \equiv 2 \pmod 7$ $\qquad x \equiv 5 \bmod 8$

$1 \equiv 1 \pmod 5$ inverse of $1 \pmod 5$ is 1

$5x \equiv 1 \pmod 7$ inverse of $5 \pmod 7$ is 3

$3x \equiv 1 \pmod 8$ inverse of $3 \pmod 8$ is 3

$112 + 240 + 525 = 877 \equiv 37 \pmod{280}$

$\boxed{\text{Solutions to the system are numbers } 37 + 280k\text{, where } k \text{ is an integer}}$