① b) $256 = 2^8 \rightarrow$ binary $100000000 - 1$

$= \underbrace{1111}_{F}\ \underbrace{1111}_{F}$

a)

check $1 + 1\cdot 2 + 1\cdot 4 + 1\cdot 8 = 15$ ✓

c) $\times AAA = 10 + 10\cdot 16 + 10\cdot \underbrace{16^2}_{256} = 10(256+17)$

$= 2730$

② $56 = 2\cdot 25 + 6$

$25 = 4\cdot 6 + 1$ $\gcd(56,25)$

$6 = 56 - 2\cdot 25$

$1 = 25 - 4\cdot 6 = 25 - 4(56 - 2\cdot 25)$

$= -4\cdot 56 + 9\cdot 25$

$\therefore$ 56 & 25 are co-prime

Bézout.

③ $a \equiv a' \bmod m \Rightarrow \exists q \quad a - a' = m\cdot q$

$b \equiv b' \bmod m \Rightarrow \exists q' \quad b - b' = mq'$

$ab - a'b' = ab - ab' + ab' - a'b'$

$= a(b-b') + (a-a')b' = amq' + mqb'$

$= m(aq' + qb') \quad \therefore \quad ab \equiv a'b' \bmod m$ ☺

④ $x \equiv 3 \bmod 5$

$3x \equiv 5 \bmod 7$

$3x \equiv 4 \bmod 11$

Note: moduli 5, 7, 11 are pairwise co-prime.

$3^{-1} \equiv 5 \bmod 7$ $5 \cdot 3x \equiv 25 \bmod 7$

$x \equiv 4 \bmod 7$

Alt: $\cancel{3}x \equiv 5 \bmod 7 \equiv \cancel{12}^{\,4} \bmod 7$:)

$3^{-1} \equiv 4 \bmod 11$ $4 \cdot 3x \equiv 16 \bmod 11$

$x \equiv 5 \bmod 11$

$\cancel{3}x \equiv 4 \bmod 11$

$\equiv \cancel{15}^{\,5} \bmod 11$

New system:

$x \equiv 3 \bmod 5$

$x \equiv 4 \bmod 7$

$x \equiv 5 \bmod 11$

$m = 5 \cdot 7 \cdot 11 = 385$

i	m_i	$M_i = \frac{m}{m_i}$	$y_i = M_i^{-1} \equiv \bmod m_i$	a_i	$M_i y_i a_i$
1	5	$77 \equiv 2 \bmod 5$	3 $(3 \cdot 2 = 5 + 1)$	3	$693 \equiv 308 \bmod 385$
2	7	$55 \equiv 6 \bmod 7$	6 $(6 \cdot 6 = 5 \cdot 7 + 1)$	4	$1320 \equiv 165 \bmod 385$
3	11	$35 \equiv 2 \bmod 11$	6 $(6 \cdot 2 = 11 + 1)$	5	$1050 \equiv 280 \bmod 385$

$165 + 280 \equiv 60 \bmod 385$

Sum: $\underline{368 \bmod 385}$

⑤ $n! < n^n$ for $n > 1$

Basis: $n=2$ $\quad 2! < 2^2$ ☺
$\quad \overset{\shortparallel}{2} \quad \overset{\shortparallel}{4}$

Inductive step: Let $n > 2$.

$$n! = n\,(n-1)! < n \cdot (n-1)^{n-1} < n \cdot n^{n-1} = n^n$$ ☺

⑥ $f(0) = 3$ $\quad f(n) = -2 \cdot f(n-1)$ for $n \geq 1$

n	$f(n)$
0	3
1	$3(-2) = -6$
2	$3(-2)(-2) = 12$
3	$3(-2)(-2)(-2) = -24$
...	
n	$3\underbrace{(-2)\ldots(-2)}_{n} = 3(-2)^n$

Basis: $n=0$ $\quad f(0) = 3(-2)^0 = 3$ ☺

Inductive step: If $n \geq 1$

$$f(n) = -2\,f(n-1)$$
$$= -2 \cdot 3(-2)^{n-1} = 3 \cdot (-2)^n$$ ☺