Name: _____

Please show all work.

1. (30 pts.) Where are the following functions differentiable? Where are they analytic?

    (a) $f(z) = \operatorname{Im} z - i \operatorname{Re} z$

    (b) $f(z) = z\bar{z}$

    (c) $f(z) = \dfrac{1}{z^3 + 1}$

2. (20 pts.) Evaluate the following integrals along the given paths.

    (a) $\displaystyle\int_\gamma \operatorname{Im}\left(z^2\right)\, dz$, where γ is the straight line segment from $-i$ to -1.

    (b) $\displaystyle\int_\gamma \dfrac{e^z}{z^5 + z^2}\, dz$, where γ is $\{z : |z + 1| = 1.1\}$ traversed once counterclockwise.

3. (20 pts.) Expand the following functions in Laurent series convergent in (possibly punctured) neighborhoods of the given points. Sketch the (possibly punctured) disc of convergence. What is the radius of this disc?

    (a) $\dfrac{1}{z^3 + z}$ at 0

    (b) $\dfrac{z}{z - 1}$ at i

4. (20 pts.) Consider a plate with uniform heat conductance shaped as in the graph. Find a function that conformally maps the plate to a rectangular region. For each of the following two sets of boundary conditions, find the steady state temperature distribution in the plate.
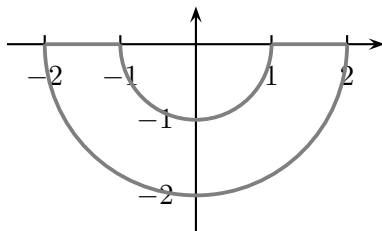
    (a) The part of the boundary on the x axis is insulated, the inner boundary arc is kept at 10° and the outer boundary arc is kept at 20°

    (b) The boundary arcs are insulated, the interval from -2 to -1 is kept at 10° and the interval from 1 to 2 is kept at 20°.

1	2	3	4	total (90)	%