

Name: _____

Please show all work.

1. (20 pts.) Let $f(x) = 2$ on the interval $(0, 2\pi)$. Let g be the *odd* periodic extension of f .
 - (a) Sketch a few periods of g .
 - (b) Find the sine Fourier series for g .
 - (c) On the same sketch as above, graph the first sine Fourier approximant.
 - (d) Express the above Fourier series in complex exponential form.
 - (e) What will the sine Fourier series converge to at $x = 0, \pi, 2\pi, 3\pi$?

2. (20 pts.) Let $f(x) = \begin{cases} 2 & \text{for } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$
 - (a) Find the Fourier transform \hat{f} of f and show that \hat{f} is real valued.
 - (b) Sketch the energy spectrum $|\hat{f}|^2$. Which range of frequency (low, medium or high) pack the most energy?

3. (20 pts.) Find the general solution of the partial differential equation $u_x + u_y = (x - 2y)u$.

4. (20 pts.) Find the vertical deflection $u(x, t)$, $0 < x < 2$ of a taut string fixed at both ends with initial position 0 and initial velocity $0.01 \sin(\pi x)$.

1	2	3	4	total (80)	%