Name: _____

Please show all work.

1. (20 pts.) Let $f(x) = 1 - x$ on $(0, 1)$. Let g be the *odd* periodic extension of f.

   (a) Sketch a few periods of g.

   (b) Find the sine Fourier series for g.

   (c) On the same sketch as above, graph the first sine Fourier approximant.

   (d) What will the sine Fourier series converge to at $x = 0, \pi, 2\pi, 3\pi$?

2. (20 pts.) Let $f(x) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ -1 & \text{for } -1 < x < 0 \\ 0 & \text{otherwise.} \end{cases}$

   (a) Find the Fourier transform $\widehat{f}$ of f and show that $\widehat{f}$ is purely imaginary.

   (b) Sketch the energy spectrum $\left|\widehat{f}\right|^2$.

3. (20 pts.) Find the *general* solution $u(x, y)$ of $y^3 u_x + x^4 u_y = 0$.

4. (20 pts.) Find the vertical deflection $u(x, t)$, $0 < x < 4$ of a taut string fixed at both ends with initial position 0 and initial velocity $0.02 \sin(2\pi x)$.

5. (20 pts.) Where are the following functions differentiable? Where are they analytic?

   (a) $f(x + iy) = 2xy + i(y^2 - x^2)$ \qquad (b) $f(z) = \overline{z}/\left|z\right|^2$

6. (20 pts.) Evaluate $\displaystyle\int_\gamma g(z)\, dz$, where

   (a) $g(z) = \overline{z}^2$ and γ is the straight line segment from -1 to i.

   (b) $g(z) = \dfrac{\cos z}{z^5 + z^2}$ and γ is $\{z : |z + 1| = 1.1\}$ traversed twice clockwise.

7. (20 pts.) Expand f in Laurent series convergent in the given regions.

   (a) $f(z) = \dfrac{z^5}{z^2 + i}$, $\quad \{z : |z| > 1\}$ \qquad (b) $f(z) = \dfrac{1}{z + 2}$, $\quad \{z : |z + i| < R\}$ $(R = ?)$

8. (20 pts.) Consider a plate with uniform heat conductance shaped as in the graph. Find a function that conformally maps the plate to a rectangular region. For each set of boundary conditions, find the steady state temperature distribution in the plate.
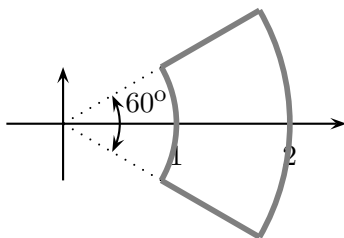
   (a) The straight parts of the boundary are insulated, the inner boundary arc is kept at $20°$ and the outer boundary arc is kept at $25°$.

   (b) The boundary arcs are insulated, the upper straight line segment is kept at $20°$ and the lower straight line segment is kept at $25°$.

1	2	3	4	5	6	7	8	total (160)	%