

Name: \_\_\_\_\_

Please show all work. If you use a theorem, name it or state it.

1. Let  $A = \begin{bmatrix} 6 & -5 \\ 13 & -10 \end{bmatrix}$ ,  $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ .

(a) Find  $e^{At}$ .

(b) Solve  $\mathbf{x}'(t) = A\mathbf{x}$ ,  $\mathbf{x}(0) = \mathbf{u}$ .

(c) Sketch  $\mathbf{x}(t)$ .

2. Same as above with  $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ .

3. Find the operator norm  $\|A\|$  for  $A$  in the preceding two problems.

4. Prove the famous inequality for the operator norm  $\|ST\| \leq \|S\| \|T\|$ .

5. Prove that matrix exponentiation preserves similarity. In other words, if  $A$  and  $B$  are similar, then so are  $e^A$  and  $e^B$ .

Hint:  $A$  and  $B$  are similar means there is an invertible  $P$  such that  $AP = PB$  (or equivalently  $A = PBP^{-1}$  or  $B = P^{-1}AP$ ).

1	2	3	4	5	total (50)