Final exam / 2014.5.6 / Algebra II / MAT 5313

Name: ______________________________

Please show all work and justify your answers.

1. Prove that for abelian groups $A \otimes (B_1 \oplus B_2) \cong (A \otimes B_1) \oplus (A \otimes B_2)$.
2. Suppose F is free $\mathbf{R}$-module on 3 generators. Prove $\mathrm{Alt}_2(F)$ is a free $\mathbf{R}$-module.
3. Suppose F is a field and G is a finite multiplicative subgroup of $F \setminus \{0\}$. Prove G is cyclic.
4. Find the sizes of conjugacy classes for S_5 and verify the class equation.
5. Same question, but for $GL(2, \mathbf{Z}_2)$.
6. Let $p(x) = x^2 + 5x + 1$, $F = \mathbf{Q}[x]/\langle p \rangle$, and $u = x + \langle p \rangle \in F$. Express u^3 and $(1-u)^{-1}$ as linear combinations of 1 and u.
7. In the above problem find the minimal polynomials of u^3 and $(1-u)^{-1}$ over $\mathbf{Q}$.
8. Find an irreducible polynomial in $\mathbf{Q}[x]$ whose Galois group over $\mathbf{Q}$ is isomorphic to S_3. Prove your assertion.

1	2	3	4	5	6	7	8	total (80)
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