

Name: \_\_\_\_\_

Please show all work and justify your answers.

Let  $K$  denote a commutative ring.

1. Give an example of a submodule of a free module that is not free. Prove your assertion.
2. Let  $\mathcal{S}$  be the set of all subgroups of the symmetric group  $S_3$ . Define  $f: S_3 \times \mathcal{S} \rightarrow \mathcal{S}$  by  $f(\sigma, S) = \sigma S \sigma^{-1}$ .
  - (a) Prove that  $f$  is a group action.
  - (b) Compute the orbits of the subgroup  $H$  generated by the involution  $(1, 2)$  and of the alternating group  $A_3$ .
  - (c) What are the normalizers  $N_{S_3}(H)$  and  $N_{S_3}(A_3)$ ? Prove your assertions.
3. The dihedral group  $\Delta_n$  is generated by  $a$  and  $d$  modulo relations  $a^n = d^2 = 1, da = a^{-1}d$ .
  - (a) Prove that the commutator subgroup  $\Delta'_n$  is cyclic.
  - (b) Find the composition factors of  $\Delta_5$ .
4. Suppose  $F$  is a field of characteristic  $\neq 2$  and  $u \in F$  satisfies  $u^2 - 2u + 3 = 0$ . Let  $s = \frac{1}{1-u}$ .
  - (a) Express  $s$  as a linear combination of 1 and  $u$ .
  - (b) Find a polynomial with rational coefficients satisfied by  $s$ .

1	2	3	4	total (40)	%

Prelim. course grade:                      %