General Topology II, MAT 5253

Midterm, March 10, 1997

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Name: ______________________________

Show all work. Box your answers.

Let $I$ denote the closed unit interval in $\mathbf{R}$ and $S^n$ denote the unit sphere in $\mathbf{R}^{n+1}$.

1. (25 pts.) Let $p\colon \mathbf{R} \to S^1 \subset \mathbf{C}$ be the map $p(x) = e^{2\pi i x}$.
   (a) Prove that $p\colon (\mathbf{R}, +) \to (S^1, \cdot)$ is a group homomorphism and that $\ker p = \mathbf{Z}$.
   (b) Let $a \in \mathbf{R}$ and define $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = x + a$. For which $a$ is $f$ fibre preserving for $p$, i.e. $p \circ f = p$?
   (c) Let $J$ be an open interval in $\mathbf{R}$. What is the maximum length of $J$ such that $p^{-1}(p(J))$ is not connected?
   (d) Consider the path $\sigma : I \to S^1$ given by $\sigma(s) = e^{-4\pi i s}$. Show that $\sigma$ is a loop with $\sigma(0) = \sigma(1) = 1$. Sketch this loop.
   (e) For the same $\sigma$ as above, find a path $\sigma' : I \to \mathbf{R}$ such that $p \circ \sigma' = \sigma$ and $\sigma'(0) = 0$. What is $\sigma'(1)$?

2. (30 pts.) Prove the following statements.
   (a) $X$ is contractible $\Rightarrow$ $X$ is path connected.
   (b) $U \subseteq \mathbf{R}^n$, $U$ is convex $\Rightarrow$ $U$ is contractible.
   (c) $\mathbf{R}^n \setminus \{0\}$ is homotopy equivalent to $S^{n-1}$.

| 1 | 2 | 3 | 4 | 5 | total (55) | % |
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