

Name: \_\_\_\_\_

Please show all work and justify your answers.

1. Prove that a continuous real-valued function on a topological space that is zero on a dense subset must be the zero function.
2. Given a family of topological spaces, pick a subset in each and prove that in general, the product of the subsets' closures is the closure of their product.
3. Suppose  $X$  is a topological space and  $A \subseteq X$ . Recall that  $A$  is a retract of  $X$  whenever there exists an onto continuous function  $X \rightarrow A$  that is identity on  $A$ .
  - (a) Prove that  $A$  is a retract of  $X$  if and only if a continuous function on  $A$  can be extended to  $X$ .
  - (b) Prove that if  $X$  is Hausdorff, then  $A$  must be closed in  $X$ .
  - (c) Prove that the unit circle in the plane is a retract of the plane punctured at the origin.
4. Given a point in a discrete space, which filters converge to that point? What happens in a trivial space?
5. Prove that the intersection of compact subsets of a Hausdorff space is compact.

1	2	3	4	5	total (50)	%

Prelim. course grade: %