General Topology I, MAT 5243
Midterm, October 22, 1996
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Name: ______________________________

Show all work. Box your answers.

1. Suppose X and Y are topological space and $f\colon X \to Y$ is a function.
   (a) Prove that if the topology of Y is indiscrete, then f is continuous.
   (b) Prove that if the topology of X is discrete, then f is continuous.

2. Suppose X, Y and Z are topological spaces, and $f : Z \to X$ and $g : Z \to Y$ are continuous functions. Let $\varphi\colon Z \to X \times Y$ be given by $\varphi(z) = (f(z), g(z))$. Prove that φ is continuous.

3. Suppose X is a topological space, I is a set, and $A_i \subseteq X$ for each $i \in I$.
   (a) Show that $\bigcup_{i\in I} \overline{A_i} \subseteq \overline{\bigcup_{i\in I} A_i}$.
   (b) Construct an example for part (a) with strict containment.

4. True or false questions, circle your choice. No justification required.
   Throughout this question, let X and Y be topological spaces.

T F (a) If $f\colon X \to Y$ and $g\colon Y \to X$ are continuous, then $f \circ g\colon Y \to Y$ is continuous.
T F (b) If A is open in X, then $\overline{A} \neq A$.
T F (c) If $A \subseteq X$, $A \neq \emptyset$, and $A \neq X$, then the boundary $\partial A \neq \emptyset$.
T F (d) If $Y \subseteq X$ and X is a metric space, then the metric topology on Y is the same as the subspace topology on Y.

1	2	3	4	total

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